Biological Modeling of Neural Networks:

Week 14 – Dynamics and Plasticity

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14.1 Reservoir computing
- Review: Random Networks
- Computing with rich dynamics

14.2 Random Networks
- stationary state
- chaos

14.3 Stability optimized circuits
- application to motor data

14.4 Synaptic plasticity
- Hebbian
- Reward-modulated

14.5 Helping Humans
- oscillations
- network states
Week 14-part 1: Review: The brain is complex

Neuronal Dynamics – Brain dynamics is complex

10,000 neurons
3 km wire

motor cortex
frontal cortex
to motor output

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frontal cortex
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Neuronal Dynamics – Brain dynamics is complex

Week 14-part 1: Review: The brain is complex

- Complex internal dynamics
- Memory
- Response to inputs
- Decision making
- Non-stationary
- Movement planning
- More than one ‘activity’ value

motor cortex

to motor output

frontal cortex
Liquid Computing/Reservoir Computing: 
exploit rich brain dynamics

Maass et al. 2002, 
Jaeger and Haas, 2004

Review: 
Maass and Buonomano,

Stream of sensory inputs

Readout 1

Readout 2
Week 14-part 1: Reservoir computing

- ‘calculate’ $V_3 + V_4$
- if-condition on $V_1$

See Maass et al. 2007

Fig. 20.1: Reservoir computing. A. A randomly connected network of integrate-and-fire neurons, receiving input from $V_1$, is characterized by a burst in $V_1$ after the burst in $V_2$. The network dynamics are represented by the connections between input and output signals. B. Neuron activity is illustrated with time series plots, showing the burst activity in $V_1$ and $V_2$, and the changes in $V_3$ and $V_4$.
Rich neuronal dynamics

Experiments of
Churchland et al. 2010
Churchland et al. 2012
See also:
Shenoy et al. 2011

Modeling
Hennequin et al. 2014,
See also:
Maass et al. 2002,
Sussillo and Abbott, 2009
Laje and Buonomano, 2012
Shenoy et al., 2011
Week 14-part 1: Rich neuronal dynamics: a wish list

- Long transients
- Reliable (non-chaotic)
- Rich dynamics (non-trivial)
- Compatible with neural data (excitation/inhibition)
- Plausible plasticity rules
Week 14 – Dynamics and Plasticity

Biological Modeling of Neural Networks:

14.1 Reservoir computing
- Review: Random Networks
- Computing with rich dynamics

14.2 Random Networks
- Rate model
- Stationary state and chaos

14.3 Hebbian Plasticity
- Excitatory synapses
- Inhibitory synapses

14.4 Reward-modulated plasticity
- Free solution

14.5 Helping Humans
- Time dependent activity
- Network states
Week 14-part 2: Review: microscopic vs. macroscopic
Homogeneous network:
- each neuron receives input from k neurons in network
- each neuron receives the same (mean) external input

Week 14-part 2: Review: Random coupling

excitation

inhibition
Stochastic spike arrival:
- excitation, total rate $R_e$
- inhibition, total rate $R_i$

Firing times:
- Threshold crossing

Synaptic current pulses:
$$\tau \frac{d}{dt} u = -(u - u_{eq}) + R \left\{ \sum_{k,f} q_e \delta(t - t_k^f) - \sum_{k',f'} q_i \delta(t - t_{k',f'}) \right\}$$

Langevin equation,
- Ornstein Uhlenbeck process
  $\rightarrow$ Fokker-Planck equation
\[
\frac{d}{dt} r_i = -r_i + F(\sum_j w_{ij} r_j)
\]

Fixed point with \(F(0)=0\) \(\Rightarrow\) 

\[r_i = 0\]

stable \quad \text{unstable}

Suppose:

\[1 = F'(0) = \frac{d}{dx} F(x = 0)\]

Suppose 1 dimension

\[
\frac{d}{dt} x = -x + F(wx)
\]
Exercise 1: Stability of fixed point

\[ \frac{d}{dt} x = -x + F(wx) \]

Fixed point with \( F(0) = 0 \) \( \rightarrow \) \( x = 0 \)

Suppose:

\[ 1 = F'(0) = \frac{d}{dx} F(x = 0) \]

Suppose 1 dimension

\[ \frac{d}{dt} x = -x + F(wx) \]

Calculate stability, take \( w \) as parameter
Blackboard: Two dimensions!

\[ \frac{dr_i}{dt} = -r_i + F\left( \sum_j w_{ij} r_j \right) \]

Fixed point with \( F(0) = 0 \) \( \Rightarrow \)

\[ r_i = 0 \]

Suppose:

\[ 1 = F'(0) = \frac{d}{dx} F(x = 0) \]

Suppose 1 dimension:

\[ \frac{d}{dt} x = -x + F(wx) \]
**Week 14-part 2: Dynamics in RANDOM Rate Networks**

\[ \frac{d}{dt} r_i = -r_i + F \left( \sum_j w_{ij} r_j \right) \]

Fixed point:

\[ r_i = 0 \]

**Random, 10 percent connectivity**

\[ 1 = F'(0) = \frac{d}{dx} F(x = 0) \]

**Chaotic dynamics:**

Sompolinksy et al. 1988

(and many others: Amari, ...)

stable
\[ \text{Re}(\lambda) < 1 \]

unstable
\[ \text{Re}(\lambda) > 1 \]
Unstable dynamics and Chaos

\[ \frac{d}{dt} r_i = -r_i + F\left(\sum_j w_{ij} r_j\right) + \xi_i(t) \]

Rajan and Abbott, 2006  
Image: Ostojic, Nat.Neurosci, 2014

\( \text{Re}(\lambda) < 1 \)

\[
\frac{d}{dt} u_i = -u_i + \sum_j \sum_f w_{ij} \delta(t - t^f_j)
\]

Firing times: Threshold crossing

Image: Ostojic, Nat.Neurosci, 2014
Switching/bursts $\rightarrow$ long autocorrelations:
Rate chaos
\[ \text{Re}(\lambda) > 1 \]
Week 14-part 2: Rich neuronal dynamics: a wish list

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- Plausible plasticity rules
Reservoir computing
- Review: Random Networks
- Computing with rich dynamics

Random Networks
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- chaos

Stability optimized circuits
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Synaptic plasticity
- Hebbian
- Reward-modulated

Helping Humans
- time dependent activity
- network states
Week 14-part 3: Plasticity-optimized circuit

Optimal control of transient dynamics in balanced networks supports generation of complex movements

Hennequin et al. 2014,
Random stability - optimized circuit (SOC)

Random connectivity
Random Plasticity-optimized circuit

a) strong, intricate connections

b) unstable

real part

imag. part

Random postsyn.

unstable

presyn.
Week 14-part 3: Random Plasticity-optimized circuit

\[ a_1 = \text{slowest} \]
\[ = \text{most amplified} \]

slope 1/linear theory

F-I curve of rate neuron
slope 1/linear theory

\[ a_1 = \text{slowest} \]
\[ = \text{most amplified} \]
Optimal control of transient dynamics in balanced networks supports generation of complex movements.

Hennequin et al. NEURON 2014,
Week 14-part 3: Application to motor cortex: data and model

Churchland et al. 2010/2012

Hennequin et al. 2014
Week 14-part 3: Random Plasticity-optimized circuit

Comparison: weak random

Hennequin et al. 2014
Week 14-part 3: Stability optimized SPIKING network

Classic sparse random connectivity (Brunel 2000)

Random connections, fast

Stabilizy-optimized random connections

‘distal’ connections, slow,

(Branco&Hausser, 2011) structured

Overall:

20% connectivity

12000 excitatory LIF = 200 pools of 60 neurons

3000 inhibitory LIF = 200 pools of 15 neurons
Classic sparse random connectivity (Brunel 2000)

Spontaneous firing rate

Week 14-part 3: Stability optimized SPIKING network

Hennequin et al. 2014

7: Transient dynamics in a spiking SOC. (a) The network is initialized in a mixture of its topological motifs. Spontaneous activity is shown in the first two conditions, preparatory activity for the movement in the last condition. For each condition, the trials are grouped in different colors, and the corresponding mean firing rate is shown at the bottom. (b) CV ISI for 0.1 mV at 100 ms. (c) Spike rate histograms at the same time. (d) Spike raster for the single neuron with different initial conditions. (e) jPC1 projection of the first 100 ms with different initial conditions.
Week 14-part 3: Stability optimized SPIKING network

Classic sparse random connectivity (Brunel 2000)

Hennequin et al. 2014
Week 14-part 3: Rich neuronal dynamics: a result list

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Hebbian Learning
= all inputs/all times are equal

\[ \Delta w_{ij} \propto F(\text{pre}, \text{post}) \]
Week 14-part 4: STDP = spike-based Hebbian learning

Pre-before-post: potentiation of synapse

Pre-after-post: depression of synapse
Modulation of Learning = Hebb+ confirmation

Functional Postulate
Useful for learning the important stuff

$$\Delta w_{ij} \propto F(pre, post, CONFIRM)$$

Many models (and experiments) of synaptic plasticity do not take into account Neuromodulators.

Consolidation of Learning

Success/reward

Confirmation
- Novel
- Interesting
- Rewarding
- Surprising

Neuromodulators
dopamine/serotonin/Ach

‘write now’
to long-term memory

Crow (1968),
Fregnac et al (2010),
Izhikevich (2007)
Plasticity

Stability-optimized circuits
- here: algorithmically tuned

BUT
- replace by inhibitory plasticity

→ avoids chaotic blow-up of network
→ avoids blow-up of single neuron (detailed balance)
→ yields stability optimized circuits

Vogels et al., Science 2011
Plasticity

Readout
- here: algorithmically tuned

BUT
- replace by 3-factor plasticity rules

Success signal

Izhikevich, 2007
Fremaux et al., 2012
Dopamine emits in response to success, which is encoded as the difference between actual reward and expected reward.

**Dopamine-emitting neurons:**
- Schultz et al., 1997
- Izhikevich, 2007
- Fremaux et al., 2012

**Success signal**
Week 14-part 4: Plasticity modulated by reward
Week 14-part 4: STDP = spike-based Hebbian learning

Pre-before post: potentiation of synapse
**Fig. 19.16:** Dopamine-modulated Hebbian learning. **A.** An STDP protocol normally gives rise to long-term potentiation (pre-before-post, solid black line as in Fig. 19.4D). However, if dopamine receptors are blocked, no change occurs (Schematic representation of experiments in Pawlak and Kerr (2008)). **B.** The STDP window in a control situation (dashed line and filled data points) changes if additional extracellular dopamine is present (solid lines, large open squares); adapted from Zhang et al. (2009).
How can the readouts encode movement?
Week 14-part 5: Population vector coding

Population vector coding of movements

Schwartz et al. 1988
• 70’000 synapses
• 1 trial = 1 second
• Output to trajectories via population vector coding
• Single reward at the END of each trial based on similarity with a target trajectory

Fremaux et al., J. Neurosci. 2010
Week 14-part 4: Learning movement trajectories

Fremaux et al. J. Neurosci. 2010
**Week 14-part 4:** Plasticity can tune the network and readout

**Hebbian STDP**
- inhibitory connections, tuned by 2-factor STDP, for stabilization  
  *Vogels et al. 2011*

**Reward-modulated STDP** for movement learning
- Readout connections, tuned by 3-factor plasticity rule

**Success signal**

**Fremaux et al. 2012**
Last Lecture TODAY

Exam:
- written exam 17. 06. 2014 from 16:15-19:00
- miniprojects counts 1/3 towards final grade

For written exam:
- bring 1 page A5 of own notes/summary
- HANDWRITTEN!
Nearly the end: what can I improve for the students next year?

- Integrated exercises?
- Quizzes?
- Miniproject?
- Overall workload? (4 credit course = 6hrs per week)
- Background/Prerequisites?
  - Physics students
  - SV students
  - Math students
- Slides? videos?