Week 3 – Reducing detail: Two-dimensional neuron models

Wulfram Gerstner
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3.1 From Hodgkin-Huxley to 2D
- Overview: From 4 to 2 dimensions
- MathDetour 1: Exploiting similarities
- MathDetour 2: Separation of time scales

3.2 Phase Plane Analysis
- Role of nullclines

3.3 Analysis of a 2D Neuron Model
- constant input vs pulse input
- MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models
next week!
3.1. Review: Hodgkin-Huxley Model

The Hodgkin-Huxley model is a mathematical model that describes the ionic mechanisms underlying the initiation and propagation of action potentials in neurons. It is based on the Hodgkin-Huxley equations, which are a set of nonlinear differential equations that describe the time evolution of the membrane potential of a neuron. The model consists of four equations, one for each of the ionic currents that contribute to the membrane potential:

\[ \tau \frac{du}{dt} = g_{Na} m^3 h (V - E_{Na}) - g_{K} n^4 (V - E_{K}) - g_{Cl} n^4 (V - E_{Cl}) - I(t) \]

where:
- \( u \) is the membrane potential,
- \( \tau \) is the time constant,
- \( g_{Na} \), \( g_{K} \), and \( g_{Cl} \) are the conductances of sodium, potassium, and chloride ions, respectively,
- \( m \), \( h \), and \( n \) are the activation variables for the sodium, potassium, and chloride channels, respectively,
- \( E_{Na} \), \( E_{K} \), and \( E_{Cl} \) are the reversal potentials for sodium, potassium, and chloride ions, respectively,
- \( I(t) \) is the membrane current.

The Hodgkin-Huxley model is a key component of compartmental models used in neuroscience to study the electrical properties of neurons and neural networks.
Neuronal Dynamics – 3.1 Review: Hodgkin-Huxley Model

Week 2:
Cell membrane contains
- ion channels
- ion pumps

Dendrites (week 4):
Active processes?

assumption:
passive dendrite → point neuron
spike generation

Week 2:
Cell membrane contains
- ion channels
- ion pumps

-70mV

Ions/proteins

Ca$^{2+}$

Na$^{+}$

K$^{+}$
Neuronal Dynamics – 3.1. Review : Hodgkin-Huxley Model

\[ \Delta u = u_1 - u_2 = \frac{-kT}{q} \ln \frac{n(u_1)}{n(u_2)} \]

Reversal potential

 Ion pumps \(\rightarrow\) concentration difference \(\Leftrightarrow\) voltage difference
Neuronal Dynamics – 3.1. Review: Hodgkin-Huxley Model

Hodgkin and Huxley, 1952

4 equations = 4D system

\[
C \frac{du}{dt} = -g_{Na} m^3 h(u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t)
\]

\[
\frac{dh}{dt} = \frac{h_n - h(u)}{\tau_n(u)}
\]

\[
\frac{dm}{dt} = \frac{m_n - m(u)}{\tau_m(u)}
\]
Can we understand the dynamics of the HH model?
- mathematical principle of Action Potential generation?
- constant input current vs pulse input?
- Types of neuron model (type I and II)? (next week)
- threshold behavior? (next week)

→ Reduce from 4 to 2 equations

Type I and type II models
Can we understand the dynamics of the HH model?

→ Reduce from 4 to 2 equations
A - A biophysical point neuron model with 3 ion channels, each with activation and inactivation, has a total number of equations equal to

[ ] 3 or
[ ] 4 or
[ ] 6 or
[ ] 7 or
[ ] 8 or more
Toward a two-dimensional neuron model

-Reduction of Hodgkin-Huxley to 2 dimension
  -step 1: separation of time scales
  -step 2: exploit similarities/correlations
Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

\[ C \frac{du}{dt} = -g_{Na} m^3 h(u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t) \]

1) dynamics of \( m \) are fast

\[ \tau_m(u) \]

\[ \tau_h(u) \]

\[ \tau_n(u) \]

\[ m(t) = m_0(u(t)) \]
Two coupled differential equations

\[ \tau_1 \frac{dx}{dt} = -x + h(y) \]
\[ \tau_2 \frac{dy}{dt} = f(y) + g(x) \]

Separation of time scales

\[ \tau_1 \ll \tau_2 \rightarrow x = h(y) \]

Reduced 1-dimensional system

\[ \tau_2 \frac{dy}{dt} = f(y) + g(h(y)) \]

\[
C \frac{du}{dt} = -g_{Na} m^3 h(u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t)
\]

\[
\begin{align*}
\frac{dm}{dt} &= -\frac{m - m_0(u)}{\tau_m(u)} \\
\frac{dh}{dt} &= -\frac{h - h_0(u)}{\tau_h(u)} \\
\frac{dn}{dt} &= -\frac{n - n_0(u)}{\tau_n(u)}
\end{align*}
\]

1) dynamics of \( m \) are fast
2) dynamics of \( h \) and \( n \) are similar

\[m(t) = m_0(u(t))\]
Reduction of Hodgkin-Huxley Model to 2 Dimension

-step 1:
  separation of time scales

-step 2:
  exploit similarities/correlations

Now!
Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

\[ C \frac{du}{dt} = -g_{Na} m^3 h(u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t) \]

2) dynamics of \( h \) and \( n \) are similar

MathDetour

\[ 1 - h(t) = a n(t) \]
dynamics of $h$ and $n$ are similar

$1 - h(t) = an(t)$

Math. argument

Neuronal Dynamics – Detour 3.1. Exploit similarities/correlations
dynamics of $h$ and $n$ are similar

$$1 - h(t) = a \, n(t)$$

at rest

$$\frac{dh}{dt} = - \frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = - \frac{n - n_0(u)}{\tau_n(u)}$$
Neuronal Dynamics – Detour 3.1. Exploit similarities/correlations

The dynamics of $h$ and $n$ are similar:

(i) Rotate coordinate system
(ii) Suppress one coordinate
(iii) Express dynamics in new variable

$$1 - h(t) = a \ n(t) = w(t)$$

$$\frac{dh}{dt} = - \frac{h - h_0(u)}{\tau_h(u)}$$
$$\frac{dn}{dt} = - \frac{n - n_0(u)}{\tau_n(u)}$$
$$\frac{dw}{dt} = - \frac{w - w_0(u)}{\tau_{eff}(u)}$$
Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

\[
C \frac{du}{dt} = -g_{Na}[m(t)]^3 h(t) (u(t) - E_{Na}) - g_K[n(t)]^4 (u(t) - E_K) - g_l(u(t) - E_l) + I(t)
\]

\[
C \frac{du}{dt} = -g_{Na} m_0(u)^3 (1-w)(u - E_{Na}) - g_K \left[ \frac{w}{a} \right]^4 (u - E_K) - g_l(u - E_l) + I(t)
\]

1) dynamics of \( m \) are fast \( \rightarrow m(t) = m_0(u(t)) \)

2) dynamics of \( h \) and \( n \) are similar \( \rightarrow 1-h(t) = a n(t) \)

\[
\frac{dh}{dt} = - \frac{h-h_0(u)}{\tau_h(u)}
\]

\[
\frac{dn}{dt} = - \frac{n-n_0(u)}{\tau_n(u)}
\]

\[
\frac{dw}{dt} = - \frac{w-w_0(u)}{\tau_{eff}(u)}
\]
Neuronal Dynamics – 3.1. Reduction of Hodgkin-Huxley model

\[
C \frac{du}{dt} = -g_{Na} m_0(u)^3 (1-w)(u-E_{Na}) - g_K \left(\frac{w}{a}\right)^4 (u-E_K) - g_l (u-E_l) + I(t)
\]

\[
\frac{dw}{dt} = - \frac{w-w_0(u)}{\tau_{eff}(u)}
\]

\[
\tau \frac{du}{dt} = F(u(t), w(t)) + R I(t)
\]

\[
\tau_w \frac{dw}{dt} = G(u(t), w(t))
\]
NOW Exercise 1.1-1.4: separation of time scales

\[ C \frac{du}{dt} = -g_{Na} m^3 h(u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t) \]

\[
\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)} \\
\frac{dx}{dt} = -\frac{x - c(t)}{\tau} \\
\frac{dm}{dt} = -\frac{m - c(u)}{\tau_m} \\
du = f(u) - m
\]

A: 
- calculate \( x(t) \)!
- what if \( \tau \) is small?

B: 
- calculate \( m(t) \) if \( \tau \) is small!
- reduce to 1 eq.

Exercises:
1.1-1.4 now!
1.5 homework

Exerc. 9h50-10h00
Next lecture: 10h15
Biological Modeling of Neural Networks

Week 3 – part 1: Reduction of the Hodgkin-Huxley Model

3.1 From Hodgkin-Huxley to 2D
- Overview: From 4 to 2 dimensions
- MathDetour 1: Exploiting similarities
- MathDetour 2: Separation of time scales

3.2 Phase Plane Analysis
- Role of nullclines

3.3 Analysis of a 2D Neuron Model
- constant input vs pulse input
- MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models
next week!
Two coupled differential equations

\[ \tau_1 \frac{dx}{dt} = -x + c(t) \]

\[ \tau_2 \frac{dy}{dt} = f(y) + g(x) \]

Separation of time scales

\[ \tau_1 \ll \tau_2 \]

Reduced 1-dimensional system

\[ \tau_2 \frac{dy}{dt} = f(y) + g(c(t)) \]
Linear differential equation

\[ \tau_1 \frac{dx}{dt} = -x + c(t) \]
Two coupled differential equations

\[ \tau_1 \frac{dx}{dt} = -x + c(t) \]

\[ \tau_2 \frac{dc}{dt} = -c + f(x) + I(t) \]

\[ \tau_1 \ll \tau_2 \]

‘slow drive’
Neuronal Dynamics – Reduction of Hodgkin-Huxley model

\[ C \frac{du}{dt} = -g_{Na} m^3 h(u - E_{Na}) - g_K n^4 (u - E_K) - g_l (u - E_l) + I(t) \]

\[ \frac{dm}{dt} = - \frac{m - m_0(u)}{\tau_m(u)} \]

\[ \frac{dh}{dt} = - \frac{h - h_0(u)}{\tau_h(u)} \]

\[ \frac{dn}{dt} = - \frac{n - n_0(u)}{\tau_n(u)} \]

dynamics of \( m \) is fast

\[ m(t) = m_0(u(t)) \]

Fast compared to what?
Two coupled differential equations

\[ \tau_1 \frac{dx}{dt} = -x + h(y) \]
\[ \tau_2 \frac{dy}{dt} = f(y) + g(x) \]

Separation of time scales

\[ \tau_1 \ll \tau_2 \rightarrow x = h(y) \]

Reduced 1-dimensional system

\[ \tau_2 \frac{dy}{dt} = f(y) + g(h(y)) \]
**A- Separation of time scales:**

We start with two equations

\[
\tau_1 \frac{dx}{dt} = -x + y + I(t)
\]

\[
\tau_2 \frac{dy}{dt} = -y + x^2 + A
\]

[ ] If \( \tau_1 \ll \tau_2 \) then the system can be reduced to

\[
\tau_2 \frac{dy}{dt} = -y + [y + I(t)]^2 + A
\]

[ ] If \( \tau_2 \ll \tau_1 \) then the system can be reduced to

\[
\tau_1 \frac{dx}{dt} = -x + x^2 + A + I(t)
\]

[ ] None of the above is correct.

Attention

\( I(t) \) can move rapidly, therefore choice [1] not correct
Exploiting similarities:

A sufficient condition to replace two gating variables $r, s$ by a single gating variable $w$ is

- Both $r$ and $s$ have the same time constant (as a function of $u$)
- Both $r$ and $s$ have the same activation function
- Both $r$ and $s$ have the same time constant (as a function of $u$) AND the same activation function
- Both $r$ and $s$ have the same time constant (as a function of $u$) AND activation functions that are identical after some additive rescaling
- Both $r$ and $s$ have the same time constant (as a function of $u$) AND activation functions that are identical after some multiplicative rescaling
2-dimensional equation

\[
C \frac{du}{dt} = f(u(t), w(t)) + I(t)
\]

\[
\frac{dw}{dt} = g(u(t), w(t))
\]

Enables graphical analysis!

- Discussion of threshold
- Constant input current vs pulse input
- Type I and II
- Repetitive firing
Week 3 – part 1: Reduction of the Hodgkin-Huxley Model

Biological Modeling of Neural Networks

Week 3 – Reducing detail:
Two-dimensional neuron models

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3.2 Phase Plane Analysis
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3.4 Type I and II Neuron Models
next week!
Neuronal Dynamics – 3.2. Reduced Hodgkin-Huxley model

\[ C \frac{du}{dt} = -g_{Na} m_0(u)^3 (1-w)(u-E_{Na}) - g_K \left( \frac{w}{a} \right)^4 (u-E_K) - g_I (u-E_I) + I(t) \]

\[ \frac{dw}{dt} = - \frac{w-w_0(u)}{\tau_w(u)} \]

stimulus

\[ \tau \frac{du}{dt} = F(u,w) + RI(t) \]

\[ \tau_w \frac{dw}{dt} = G(u,w) \]
Neuronal Dynamics – 3.2. Phase Plane Analysis/nullclines

2-dimensional equation

\[
\tau \frac{du}{dt} = F(u, w) + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = G(u, w)
\]

Enables graphical analysis!
- Discussion of threshold
- Type I and II

u-nullcline

w-nullcline
Neuronal Dynamics – 3.2. FitzHugh-Nagumo Model

\[ \tau \frac{du}{dt} = F(u, w) + RI(t) \]
\[ = u - \frac{1}{3} u^3 - w + RI(t) \]

\[ \tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w \]

u-nullcline

w-nullcline
Neuronal Dynamics – 3.2. flow arrows

\[ \tau \frac{du}{dt} = F(u, w) + RI(t) \]

Stimulus I=0

\[ \tau_w \frac{dw}{dt} = G(u, w) \]

Consider change in small time step

Flow on nullclines

Flow in regions between nullclines
A. u-Nullclines
[ ] On the u-nullcline, arrows are always vertical
[ ] On the u-nullcline, arrows point always vertically upward
[ ] On the u-nullcline, arrows are always horizontal
[ ] On the u-nullcline, arrows point always to the left
[ ] On the u-nullcline, arrows point always to the right

B. w-Nullclines
[ ] On the w-nullcline, arrows are always vertical
[ ] On the w-nullcline, arrows point always vertically upward
[ ] On the w-nullcline, arrows are always horizontal
[ ] On the w-nullcline, arrows point always to the left
[ ] On the w-nullcline, arrows point always to the right
[ ] On the w-nullcline, arrows can point in an arbitrary direction

Take 1 minute, continue at 10:55
Neuronal Dynamics – 4.2. FitzHugh-Nagumo Model

\[ \tau \frac{du}{dt} = F(u, w) + RI(t) \]
\[ = u - \frac{1}{3} u^3 + RI(t) \]

\[ \tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w \]

change \( b_1 \)
Neuronal Dynamics – 3.2. Nullclines of reduced HH model

\[ \tau \frac{du}{dt} = F(u, w) + RI(t) \]

\[ \tau_w \frac{dw}{dt} = G(u, w) \]

Stable fixed point

Nullclines of reduced HH model

- u-nullcline
- w-nullcline

Stimulus
Neuronal Dynamics – 3.2. Phase Plane Analysis

2-dimensional equation

\[ \tau \frac{du}{dt} = F(u, w) + RI(t) \]

\[ \tau_w \frac{dw}{dt} = G(u, w) \]

Enables graphical analysis!

Important role of
- nullclines
- flow arrows

Application to neuron models
Week 3 – part 3: Analysis of a 2D neuron model

3.1 From Hodgkin-Huxley to 2D

3.2 Phase Plane Analysis
   - Role of nullclines

3.3 Analysis of a 2D Neuron Model
   - pulse input
   - constant input
   - MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models (next week)
2 important input scenarios
- Pulse input
- Constant input

2-dimensional equation
\[ \tau \frac{du}{dt} = F(u, w) + RI(t) \]
\[ \tau_w \frac{dw}{dt} = G(u, w) \]

Enables graphical analysis!
Neuronal Dynamics – 3.3. 2D neuron model: Pulse input

\[ \tau \cdot \frac{du}{dt} = F(u, w) + RI \]
\[ \tau_w \frac{dw}{dt} = G(u, w) \]
Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model: Pulse input

\[ \tau \frac{du}{dt} = F(u, w) + RI(t) = u - \frac{1}{3}u^3 - w + RI(t) \]

\[ \tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1u - w \]

Pulse input: jump of voltage
FN model with $b_0 = 0.9; b_1 = 1.0$

Pulse input: jump of voltage/initial condition

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)
Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model

Pulse input: DONE!
- jump of voltage
- ‘new initial condition’
- spike generation for large input pulses

2 important input scenarios

constant input:
- graphics?
- spikes?
- repetitive firing?

Now
Neuronal Dynamics — 3.3. FitzHugh-Nagumo Model: Constant input

\[ \tau \frac{du}{dt} = F(u, w) + RI_0 \]
\[ = u - \frac{1}{3} u^3 - w + RI_0 \]

\[ \tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w \]

Intersection point (fixed point)
- moves
- changes Stability

\[ \frac{dw}{dt} = 0 \quad \text{w-nullcline} \]
\[ \frac{du}{dt} = 0 \quad \text{u-nullcline} \]
NOW Exercise 2.1: Stability of Fixed Point in 2D

\[
\begin{align*}
\frac{du}{dt} &= \alpha u - w \\
\frac{dw}{dt} &= \beta u - w
\end{align*}
\]

- calculate *stability*
- compare

\[
\frac{dx}{dt} = -\frac{x}{\tau}
\]

Exercises:
- **2.1 now!**
- **2.2 homework**

Next lecture: 11:42
Week 3 – part 3: Analysis of a 2D neuron model

3.1 From Hodgkin-Huxley to 2D

3.2 Phase Plane Analysis
   - Role of nullcline

3.3 Analysis of a 2D Neuron Model
   - pulse input
   - constant input
   - MathDetour 3: Stability of fixed points

3.4 Type I and II Neuron Models (next week)
Neuronal Dynamics – Detour 3.3: Stability of fixed points.

\[ \tau \frac{dw}{dt} = b_0 + b_1u - w \]

\[ \frac{dw}{dt} = 0 \quad \text{w-nullcline} \]

\[ \tau \frac{du}{dt} = F(u, w) + RI_0 \]

\[ \frac{du}{dt} = 0 \quad \text{u-nullcline} \]

\[ I(t) = I_0 \]

\[ \text{stable?} \]
Neuronal Dynamics – 3.3 Detour. Stability of fixed points

2-dimensional equation

\[ \tau \frac{du}{dt} = F(u, w) + RI_0 \]

\[ \tau_w \frac{dw}{dt} = G(u, w) \]

How to determine stability of fixed point?
Neuronal Dynamics – 3.3 Detour. Stability of fixed points

\[ \tau \frac{du}{dt} = au - w + I_0 \]

\[ \tau_w \frac{dw}{dt} = cu - w \]

\[ \frac{dw}{dt} = 0 \]

\[ \frac{du}{dt} = 0 \]

stimulus

\[ I(t) = I_0 \]
Neuronal Dynamics – 3.3 Detour. Stability of fixed points

\[ \tau \frac{du}{dt} = F(u, w) + RI_0 \]

\[ \tau_w \frac{dw}{dt} = G(u, w) \]

zoom in:

stable

saddle

unstable

Math derivation now
### Neuronal Dynamics – 3.3 Detour. Stability of fixed points

\[
\tau \frac{du}{dt} = F(u, w) + RI_0
\]

\[
\tau \frac{dw}{dt} = G(u, w)
\]

**Fixed point at** \((u_0, w_0)\)

**At fixed point**

\[
0 = F(u_0, w_0) + RI_0
\]

\[
0 = G(u_0, w_0)
\]

**zoom in:**

\[x = u - u_0\]

\[y = w - w_0\]
Neuronal Dynamics – 3.3 Detour. Stability of fixed points

\[
\tau \frac{du}{dt} = F(u, w) + RI_0
\]

\[
\tau_w \frac{dw}{dt} = G(u, w)
\]

Fixed point at \((u_0, w_0)\)

At fixed point

\[
0 = F(u_0, w_0) + RI_0
\]

\[
0 = G(u_0, w_0)
\]

zoom in:

\[
x = u - u_0
\]

\[
y = w - w_0
\]

\[
\tau \frac{dx}{dt} = F_u x + F_w y
\]

\[
\tau_w \frac{dy}{dt} = G_u x + G_w y
\]

\[
\frac{d}{dt} x = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} x.
\]
Neuronal Dynamics – 3.3 Detour. Stability of fixed points

Linear matrix equation

$$\frac{d}{dt} \mathbf{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \mathbf{x}.$$ 

Search for solution

$$\mathbf{x}(t) = e^{\lambda t} \exp(\lambda t)$$

Two solution with Eigenvalues $\lambda_+, \lambda_-$

$$\lambda_+ + \lambda_- = F_u + G_w$$

$$\lambda_+ \lambda_- = F_u G_w - F_w G_u$$
Neuronal Dynamics – 3.3 Detour. Stability of fixed points

Linear matrix equation
\[ \frac{d}{dt} x = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} x \]

Search for solution
\[ x(t) = e^{\exp(\lambda t)} \]

Two solutions with Eigenvalues \( \lambda_+, \lambda_- \)
\[ \lambda_+ + \lambda_- = F_u + G_w \]
\[ \lambda_+ \lambda_- = F_u G_w - F_w G_u \]

Stability requires:
\[ \lambda_+ < 0 \quad \text{and} \quad \lambda_- < 0 \]

\[ F_u + G_w < 0 \]
\[ \text{and} \]
\[ F_u G_w - F_w G_u > 0 \]
Neuronal Dynamics – 3.3 Detour. Stability of fixed points

\[ \tau \frac{du}{dt} = au - w + I_0 \]

\[ \tau_w \frac{dw}{dt} = cu - w \]

\[ \lambda_{+/-} = \]

\[ \frac{dw}{dt} = 0 \]

\[ I(t) = I_0 \]

\[ \frac{du}{dt} = 0 \]
Now Back:

Application to our neuron model

2-dimensional equation

\[ \tau \frac{du}{dt} = F(u, w) + RI_0 \]
\[ \tau_w \frac{dw}{dt} = G(u, w) \]

Stability characterized by Eigenvalues of linearized equations

\[ \frac{d}{dt}x = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} x \]
Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model: Constant input

\[
\tau \frac{du}{dt} = F(u, w) + RI_0 = u - \frac{1}{3} u^3 - w + RI_0
\]

\[
\tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w
\]

Intersection point (fixed point)
- moves
- changes Stability

\[
\frac{dw}{dt} = 0 \quad \text{w-nullcline}
\]

\[
\frac{du}{dt} = 0 \quad \text{u-nullcline}
\]
Neuronal Dynamics – 3.3. FitzHugh-Nagumo Model: Constant input

\[
\tau \frac{du}{dt} = F(u, w) + RI_0 \\
= u - \frac{1}{3}u^3 - w + RI_0
\]

\[
\tau_w \frac{dw}{dt} = G(u, w) = b_0 + b_1 u - w
\]

Intersection point (fixed point)
- moves
- changes Stability

\[
\frac{dw}{dt} = 0 \quad \text{w-nullcline}
\]

\[
\frac{du}{dt} = 0 \quad \text{u-nullcline}
\]
FN model with $b_0 = 0.9; b_1 = 1.0; RI_0 = 2$

constant input: u-nullcline moves
limit cycle
Neuronal Dynamics – Quiz 3.4.

A. Short current pulses. In a 2-dimensional neuron model, the effect of a delta current pulse can be analyzed
- By moving the u-nullcline vertically upward
- By moving the w-nullcline vertically upward
- As a potential change in the stability or number of the fixed point(s)
- As a new initial condition
- By following the flow of arrows in the appropriate phase plane diagram

B. Constant current. In a 2-dimensional neuron model, the effect of a constant current can be analyzed
- By moving the u-nullcline vertically upward
- By moving the w-nullcline vertically upward
- As a potential change in the stability or number of the fixed point(s)
- By following the flow of arrows in the appropriate phase plane diagram
Computer exercise now

Can we understand the dynamics of the 2D model?

The END for today

Now: computer exercises

Type I and type II models

ramp input/constant input

f-I curve

f-I curve

f

I_0

f

I_0

f

I_0