Week 4: Reducing Detail – 2D models-Adding Detail

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

Week 4
– Reducing detail
- Adding detail

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Neuronal Dynamics – Review from week 3

-**Reduction of Hodgkin-Huxley to 2 dimension**

-step 1: separation of time scales

-step 2: exploit similarities/correlations

$$C\frac{du}{dt} = \underbrace{-g_{Na}[m(t)]^3 h(t)(u(t)-E_{Na})}_{I_{Na}} \underbrace{-g_K[n(t)]^4(u(t)-E_K)}_{I_K} \underbrace{-g_l(u(t)-E_l)}_{I_{leak}} + I(t)$$

$$C\frac{du}{dt} = -g_{Na}\, m_0(u)^3(1-w)(u-E_{Na}) - g_K[\frac{w}{a}]^4(u-E_K) - g_l(u-E_l) + I(t)$$

1) dynamics of *m* are fast → $m(t) = m_0(u(t))$

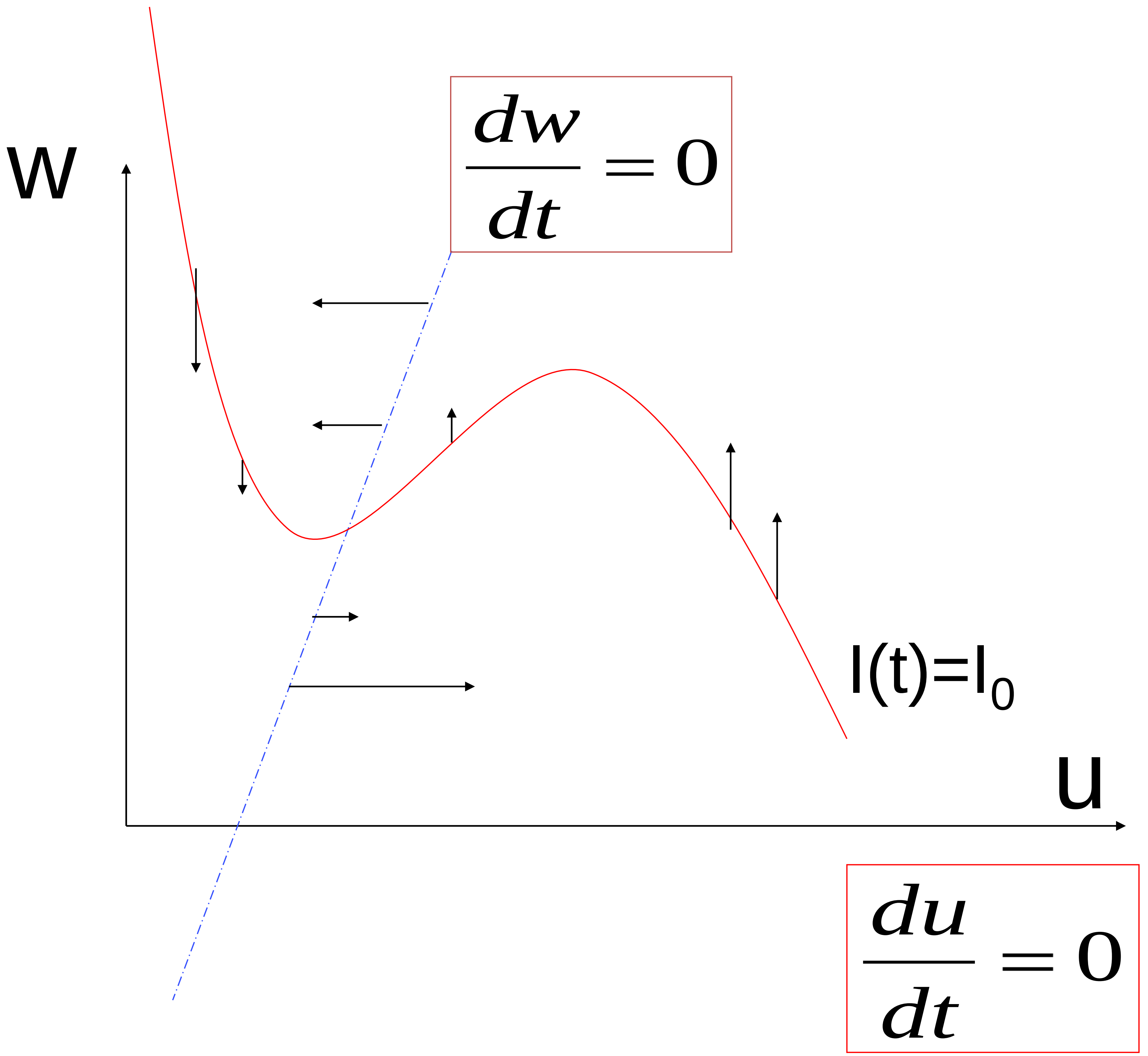
2) dynamics of *h* and *n* are similar → $\underbrace{1-h(t)}_{w(t)} = \underbrace{a\,n(t)}_{w(t)}$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$

→ $$\frac{dw}{dt} = -\frac{w-w_0(u)}{\tau_{eff}(u)}$$

Neuronal Dynamics – 4.1. Analysis of a 2D neuron model



2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Enables graphical analysis!

-Pulse input

→ AP firing (or not)

- Constant input

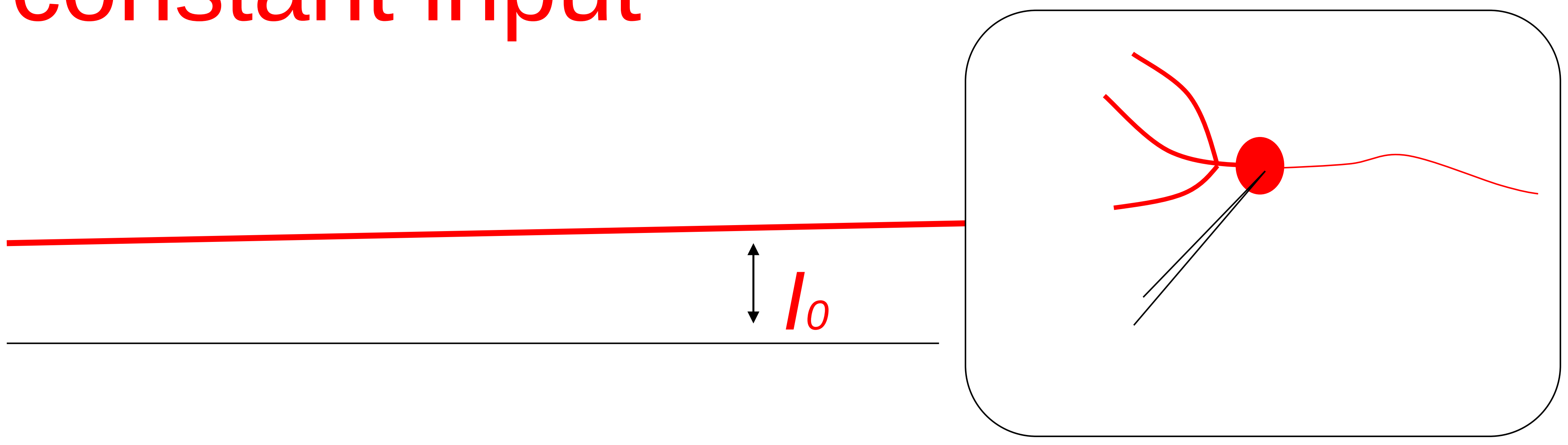
→ repetitive firing (or not)

→ limit cycle (or not)

Week 4 – part 1: Reducing Detail – 2D models

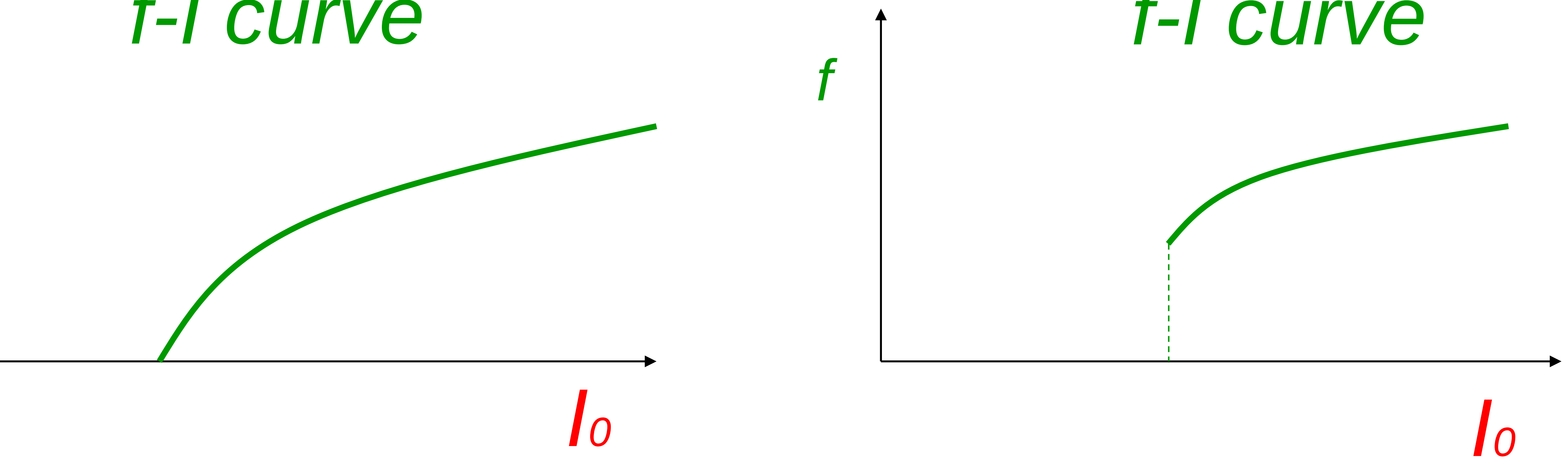
ramp input/
constant input

neuron



Type I and type II models

f-I curve *f-I curve*



√ 3.1 **From Hodgkin-Huxley to 2D**

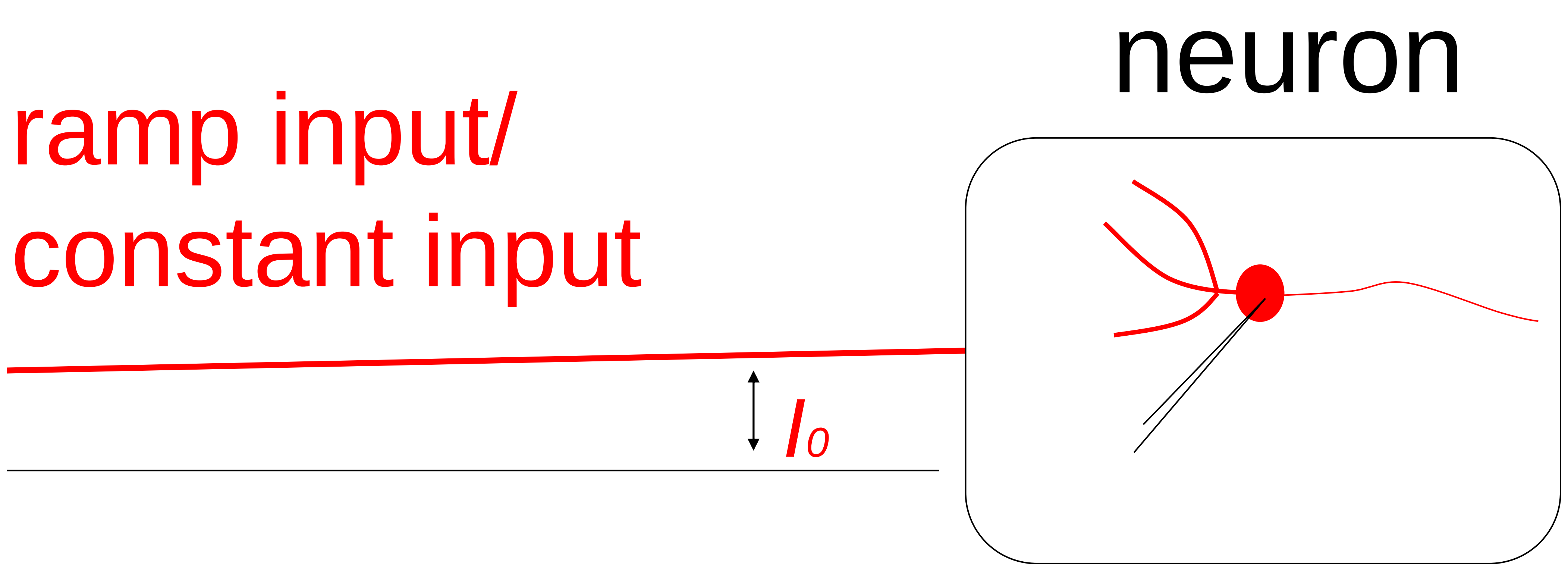
√ 3.2 **Phase Plane Analysis**

√ 3.3 **Analysis of a 2D Neuron Model**

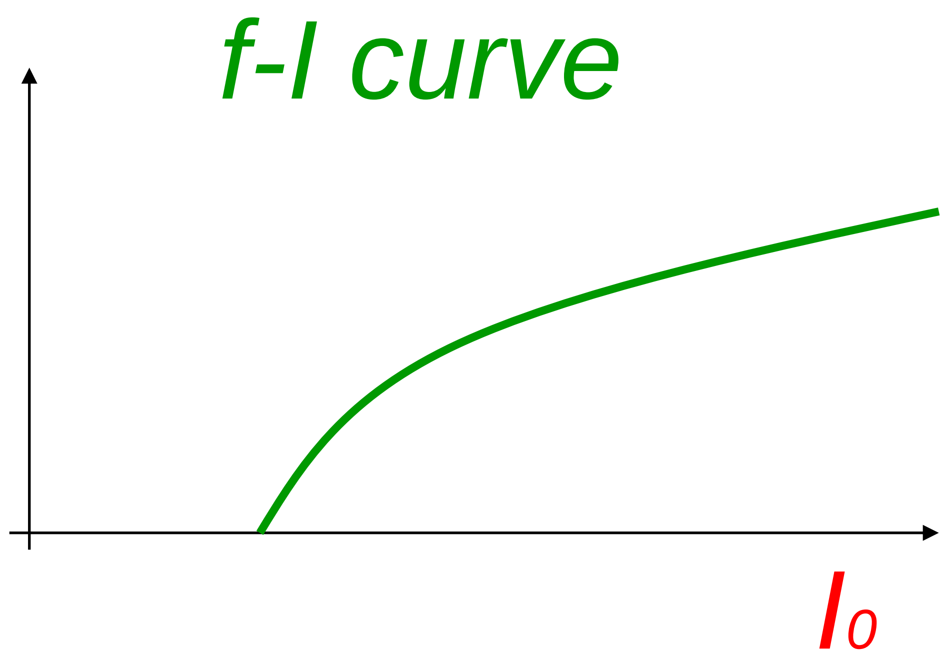
4.1 **Type I and II Neuron Models**
- limit cycles
- where is the firing threshold?
- separation of time scales

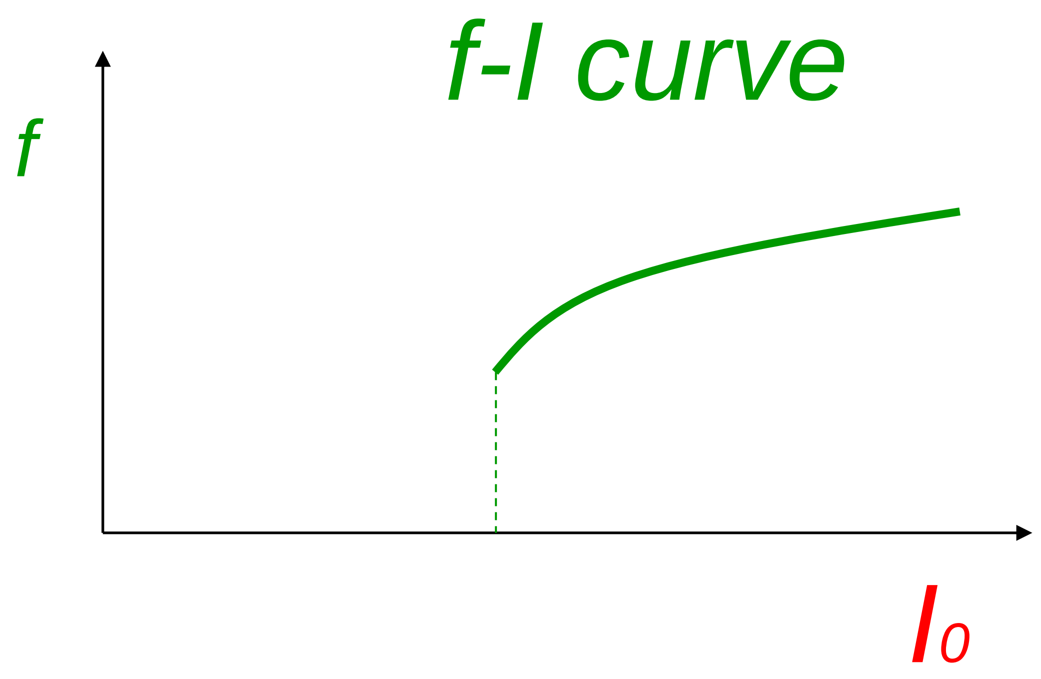
4.2. Dendrites

Neuronal Dynamics – 4.1. Type I and II Neuron Models



Type I and type II models





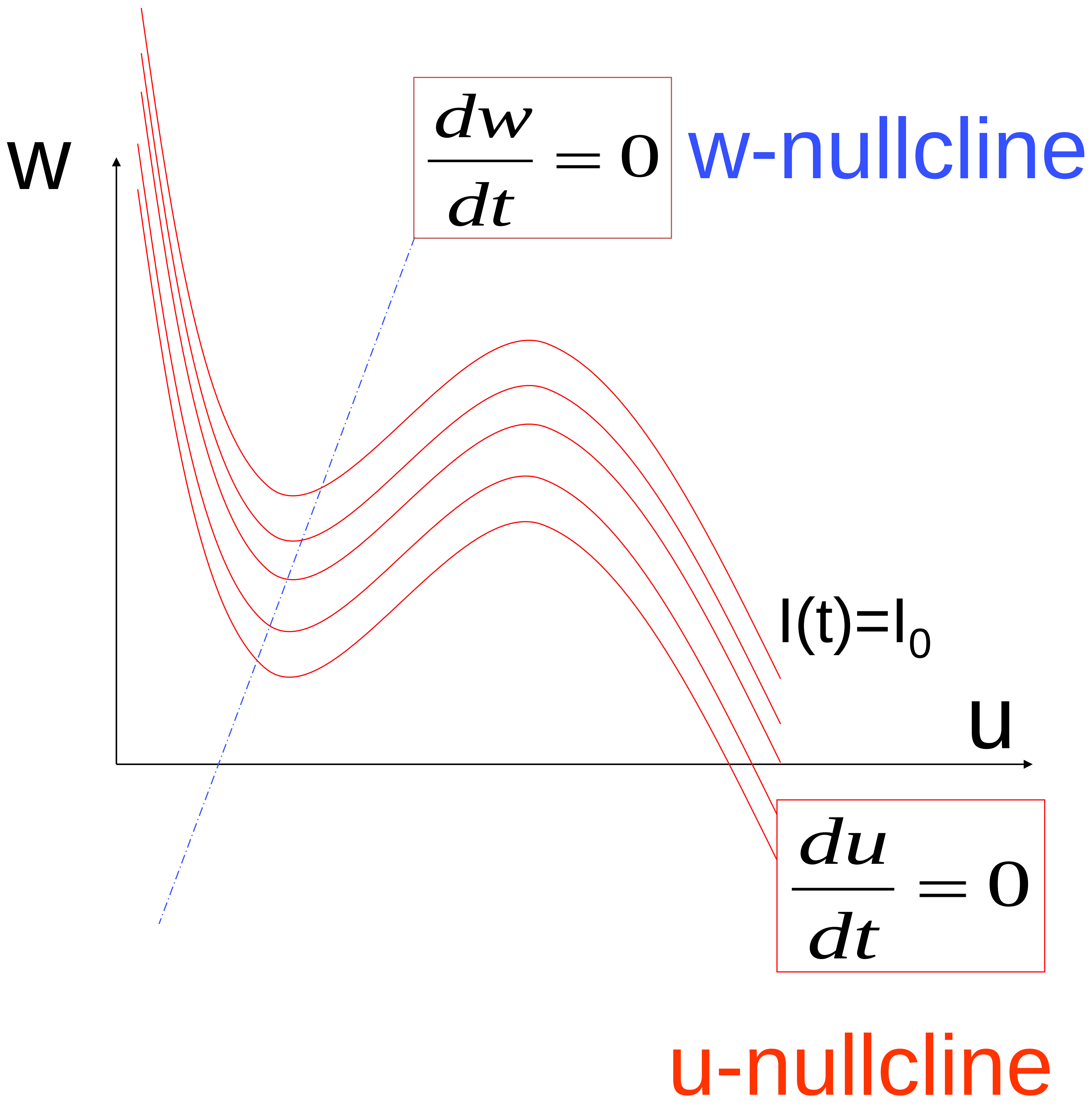
2 dimensional Neuron Models

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

apply constant stimulus I_0



FitzHugh Nagumo Model – limit cycle

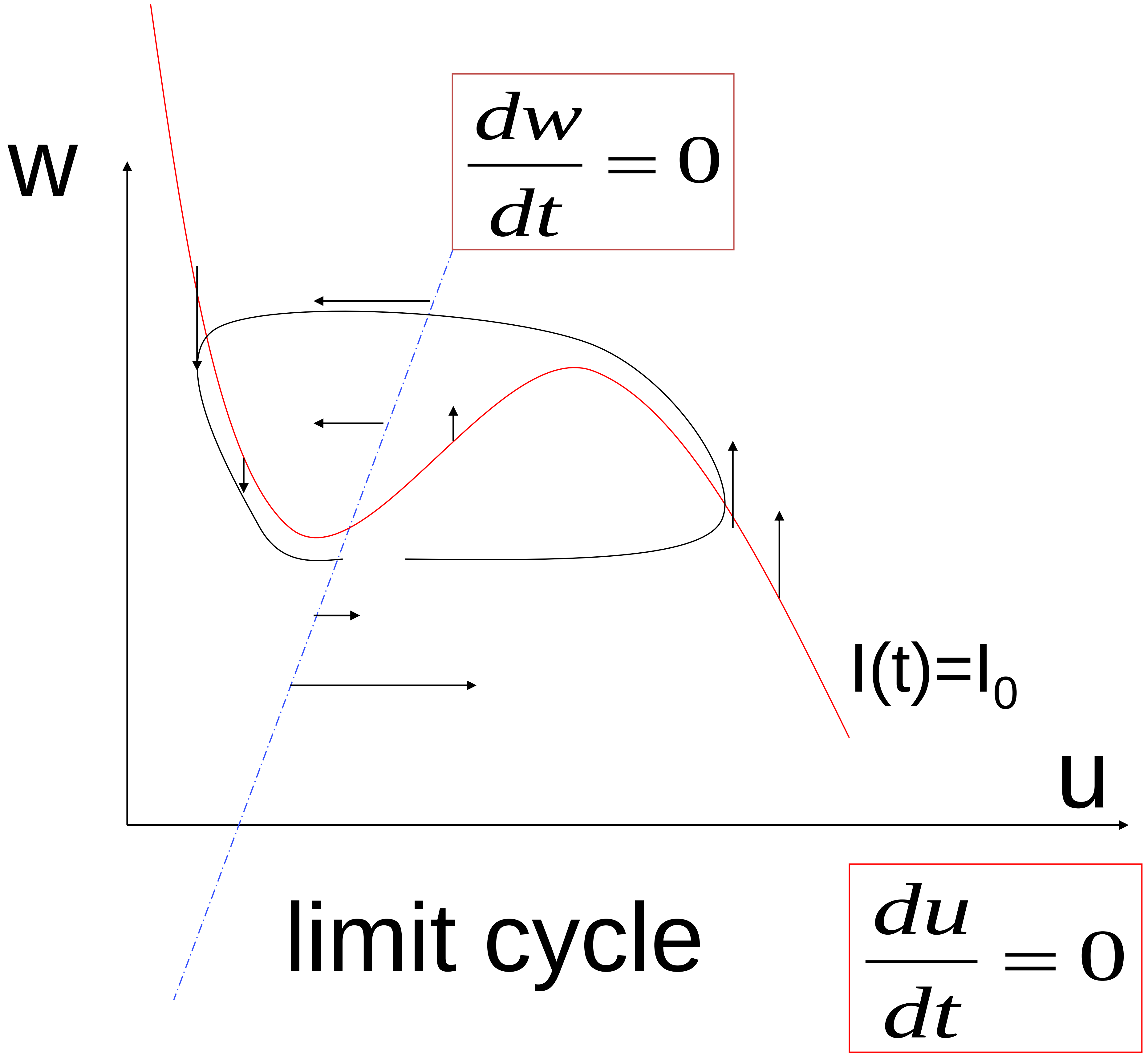
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

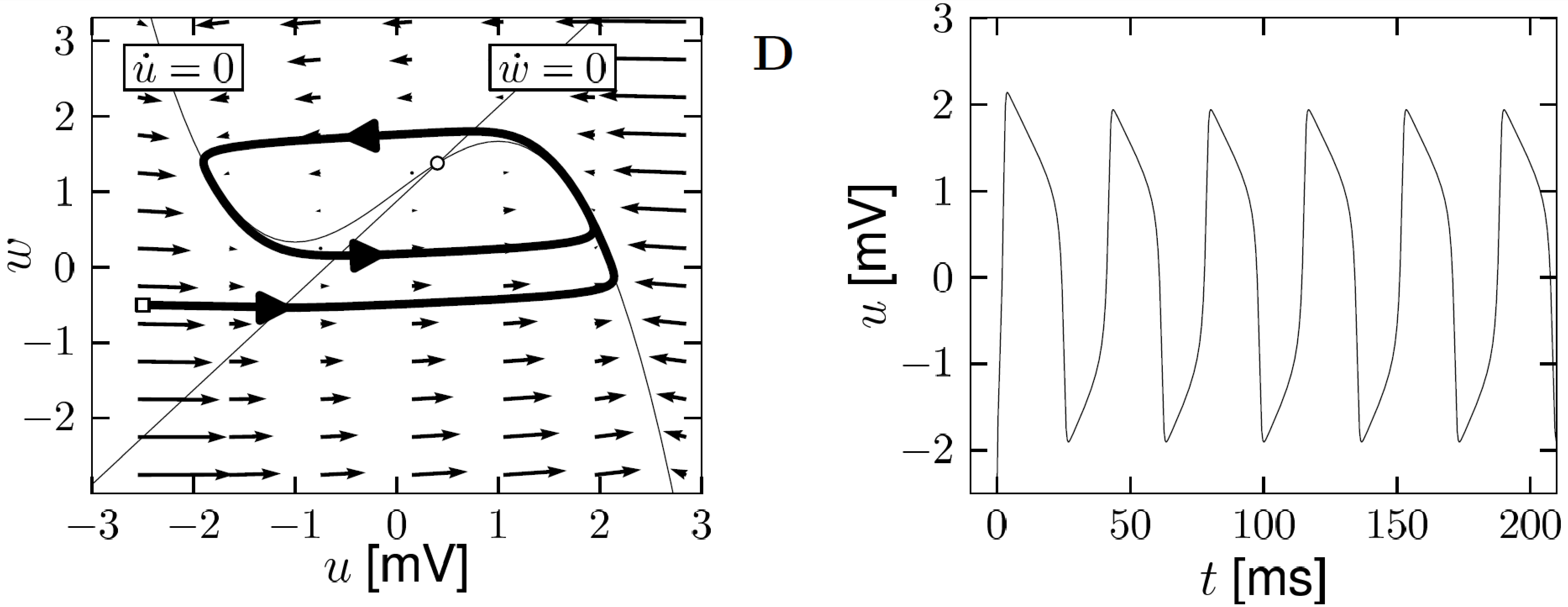
$$\tau_w \frac{dw}{dt} = G(u,w)$$

-unstable fixed point
-closed boundary
with arrows pointing inside

⟶ limit cycle



Neuronal Dynamics – 4.1. Limit Cycle



-unstable fixed point in 2D

-bounding box with inward flow

→ limit cycle *(Poincare Bendixson)*

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

In 2-dimensional equations,
a limit cycle must exist, if we can find a surface

-containing one unstable fixed point
-no other fixed point
-bounding box with inward flow
→ limit cycle *(Poincare Bendixson)*

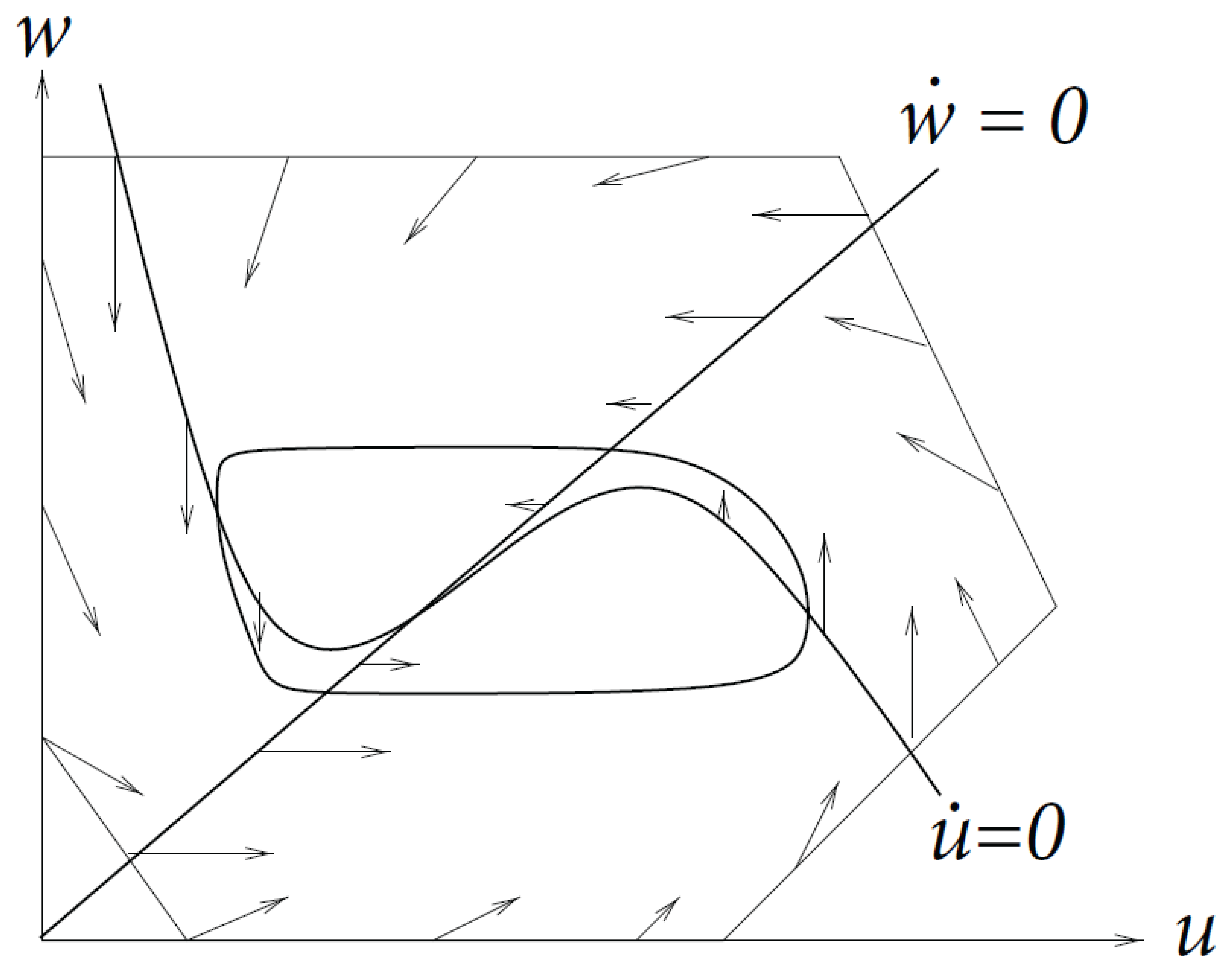


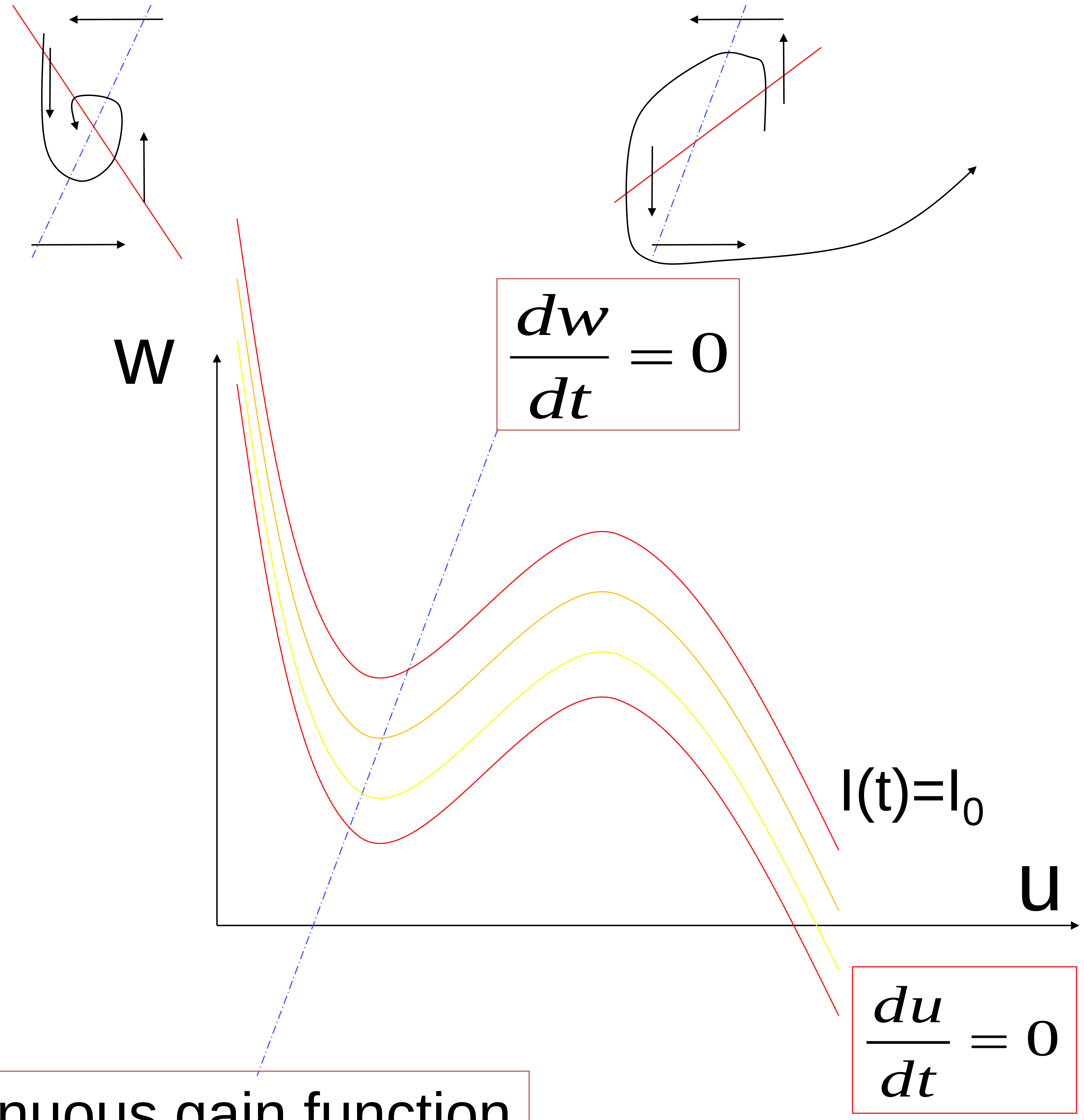
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Type II Model
constant input

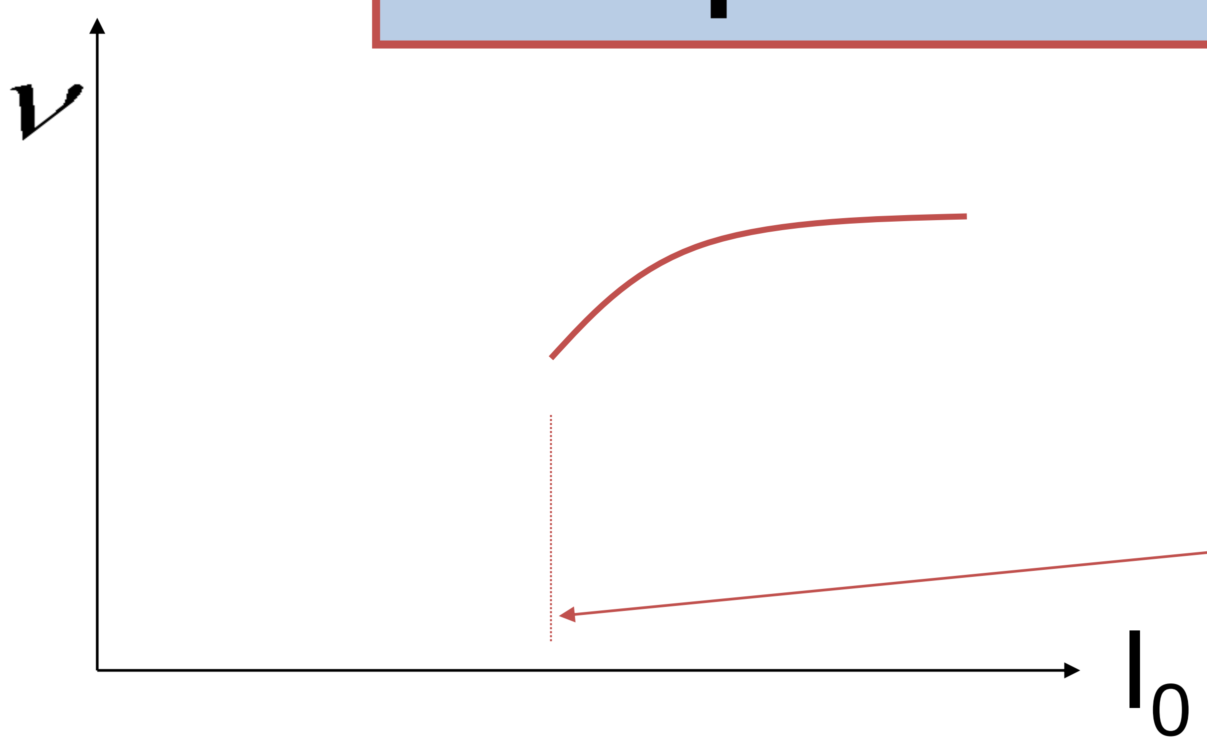
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$



Hopf bifurcation

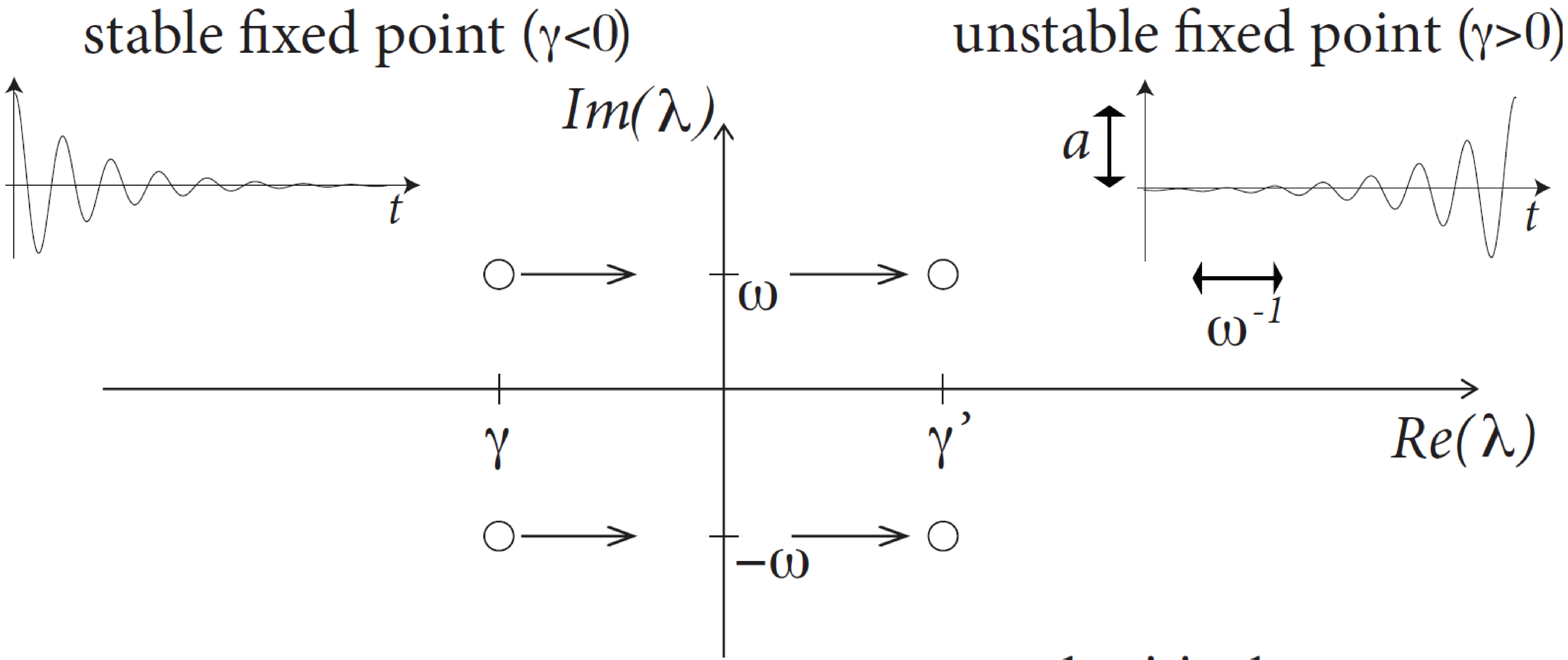


Discontinuous gain function

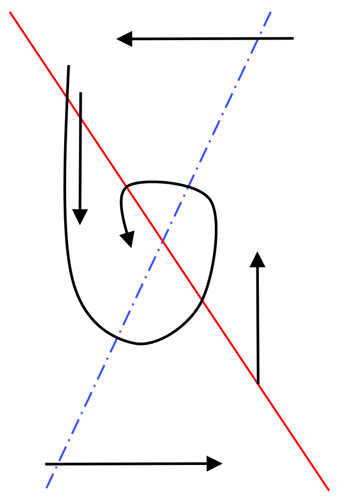
Stability lost → oscillation with finite frequency

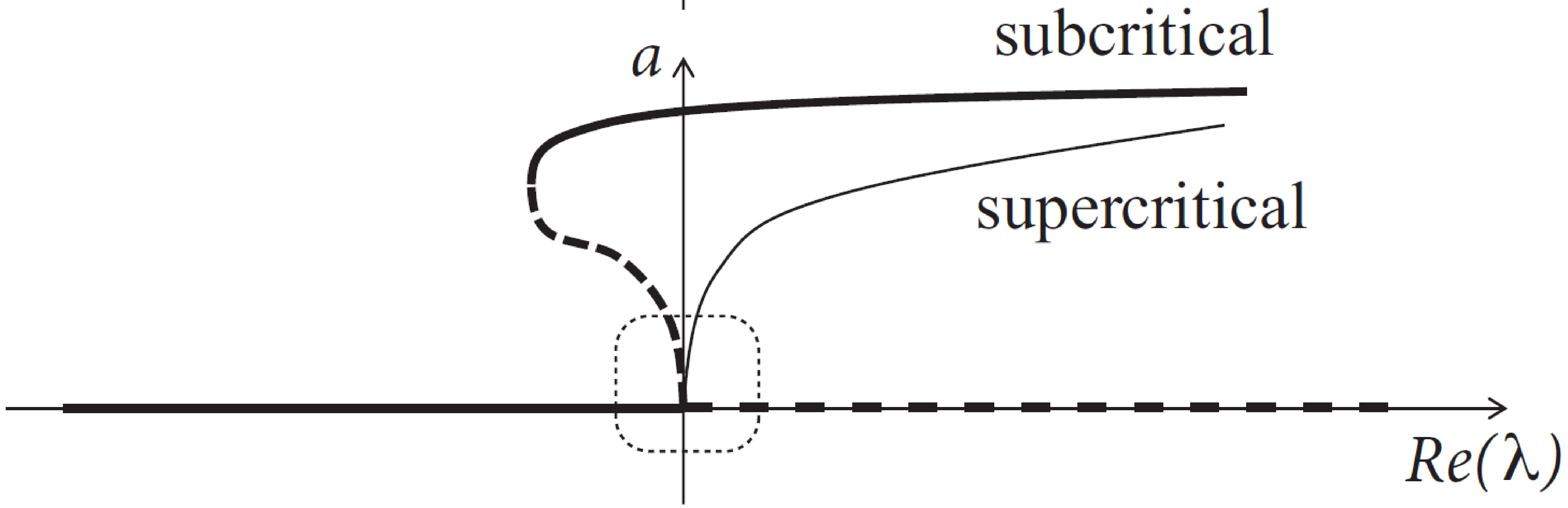
Neuronal Dynamics – 4.1. Hopf bifurcation

$$\lambda = \gamma + i\omega$$

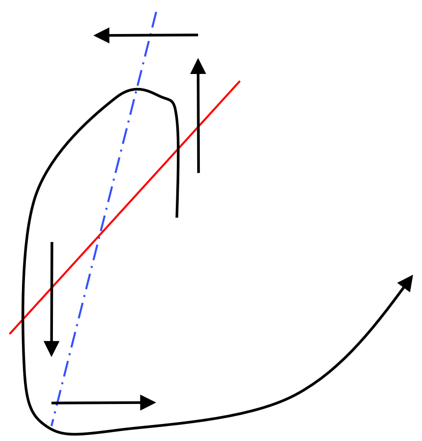


$$\gamma < 0$$

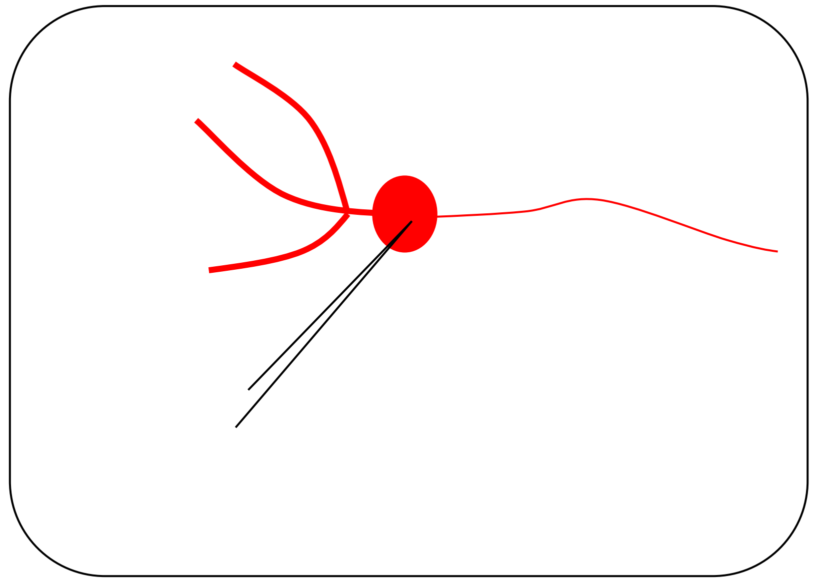




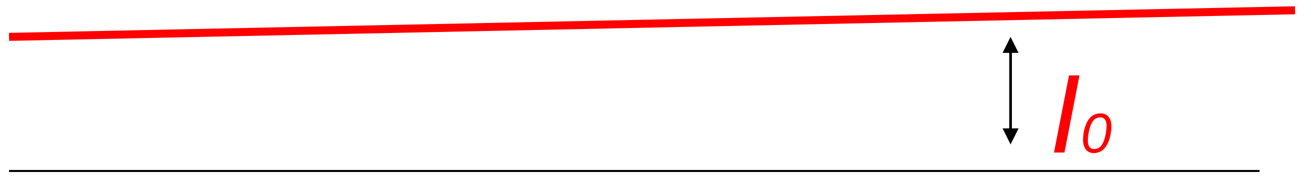
$$\gamma > 0$$



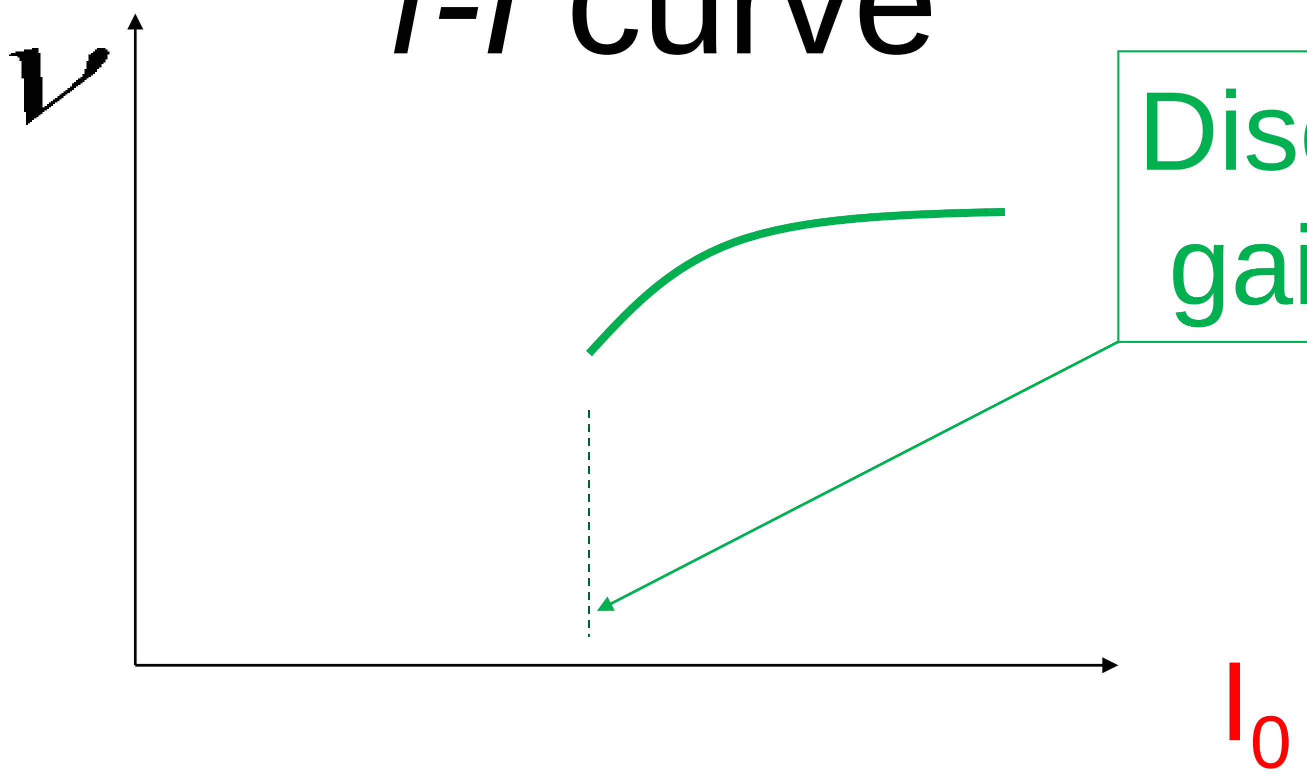
Neuronal Dynamics – 4.1. Hopf bifurcation: *f-I* -curve



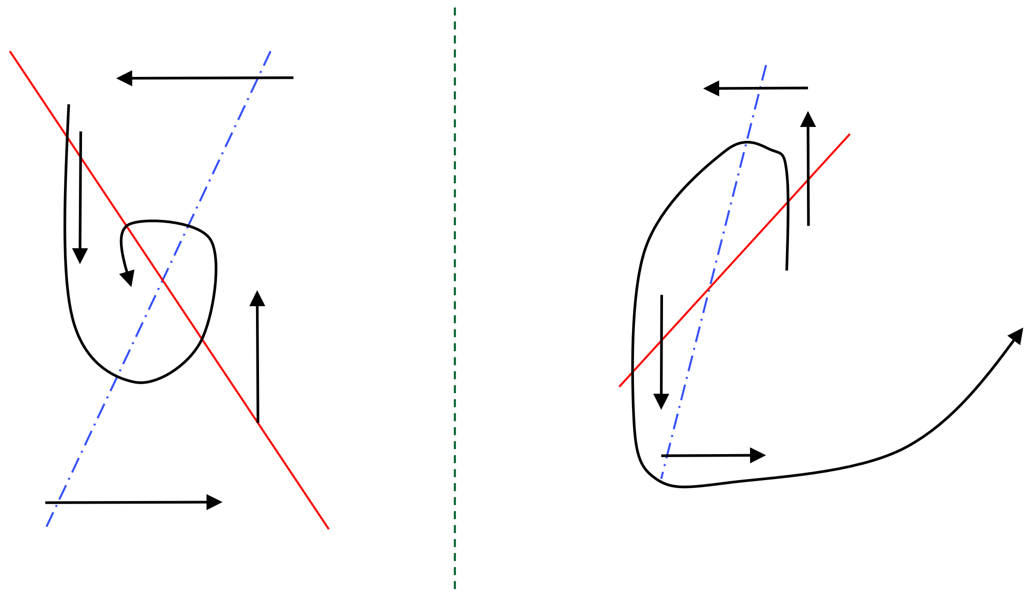
ramp input/
constant input



f-I curve



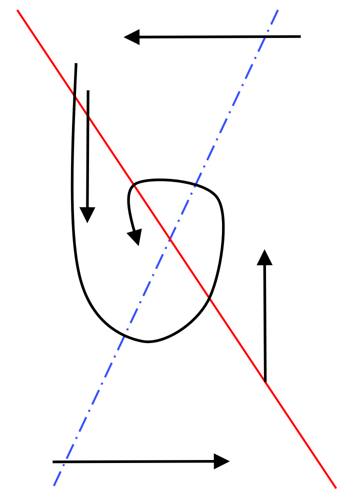
Discontinuous
gain function: Type II

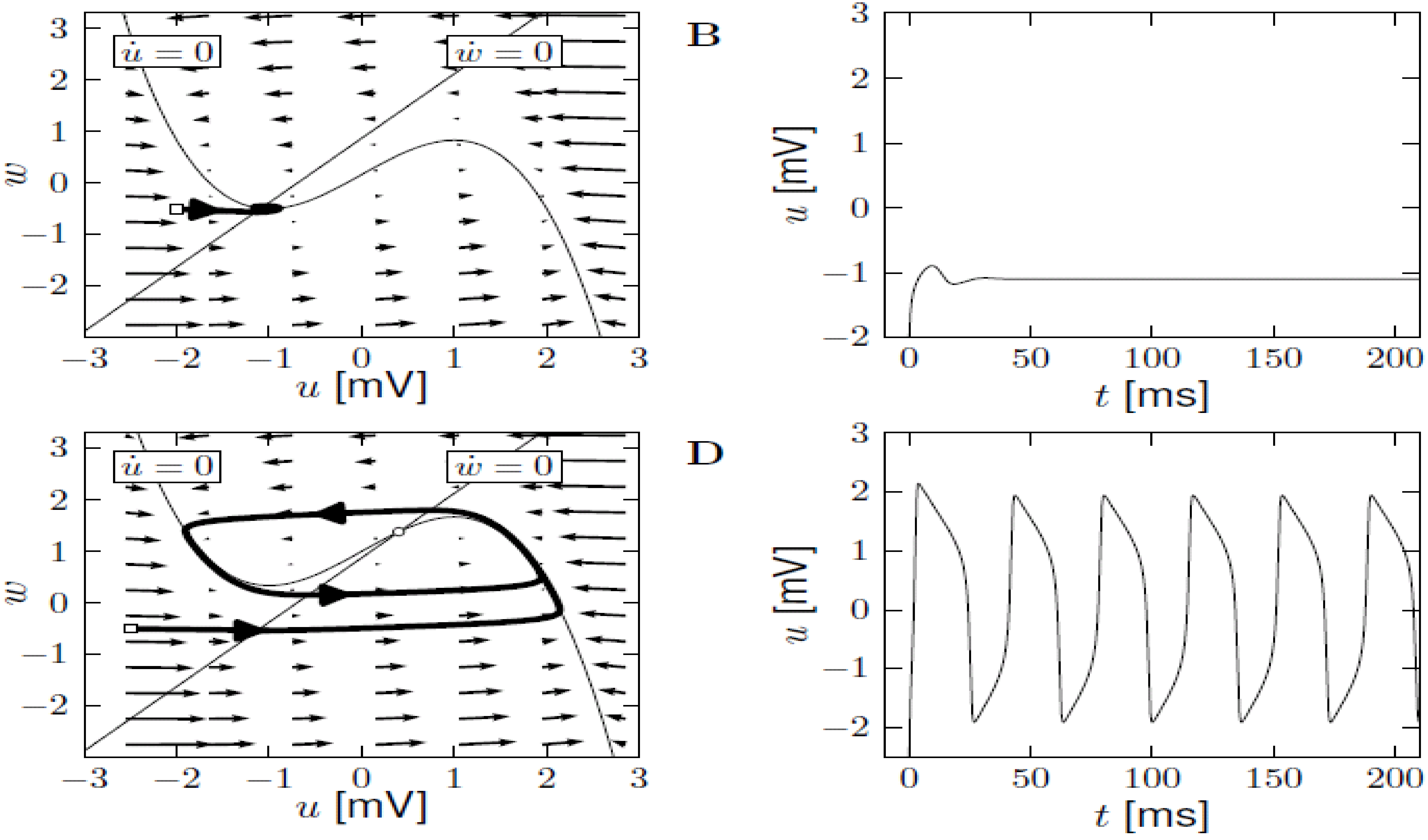


Stability lost → oscillation with finite frequency

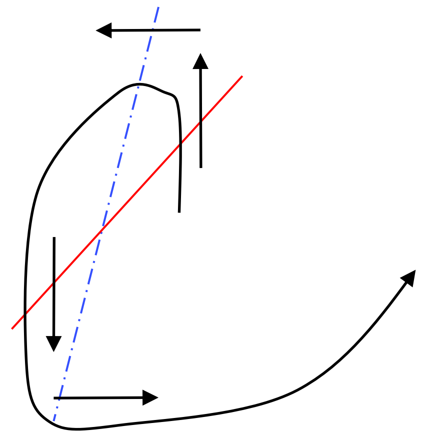
FitzHugh-Nagumo: type II Model – Hopf bifurcation

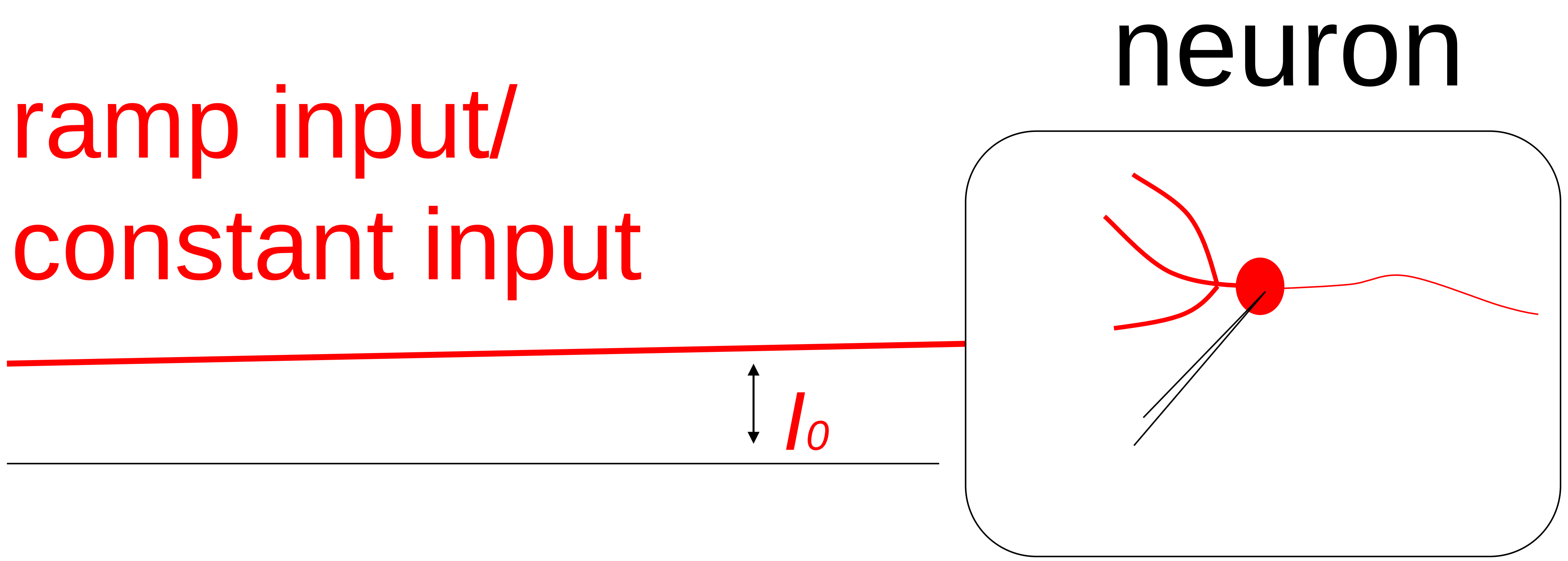
I=0





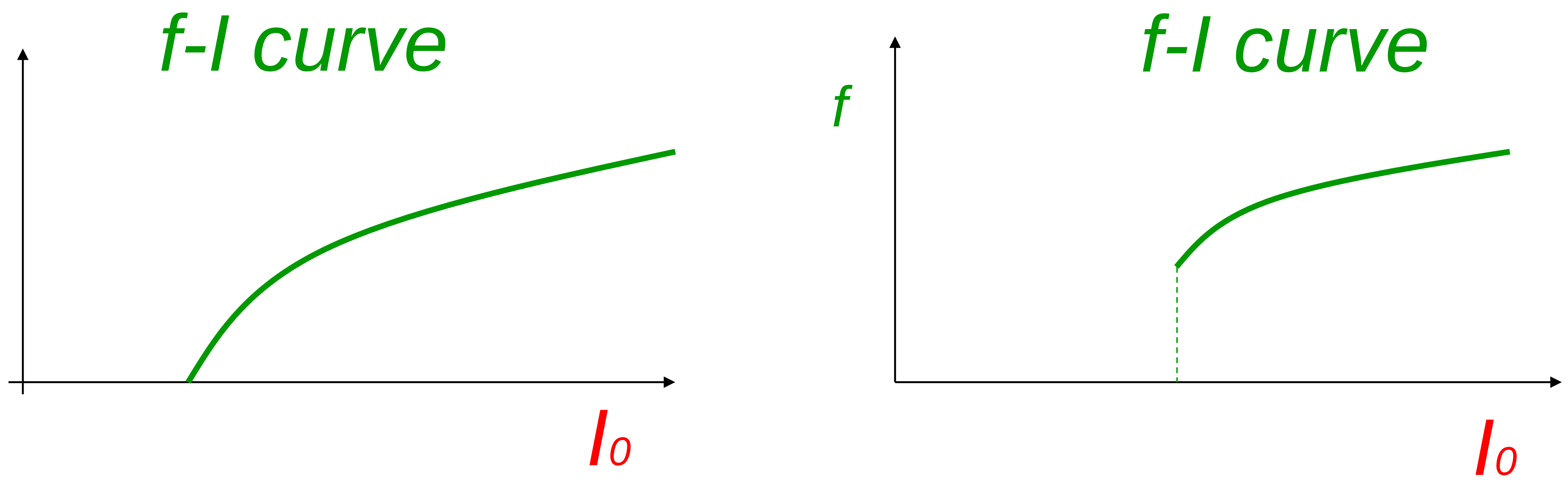
I>I$_c$





Now:
Type I model

Type I and type II models



Neuronal Dynamics – 4.1. Type I and II Neuron Models

type I Model: 3 fixed points

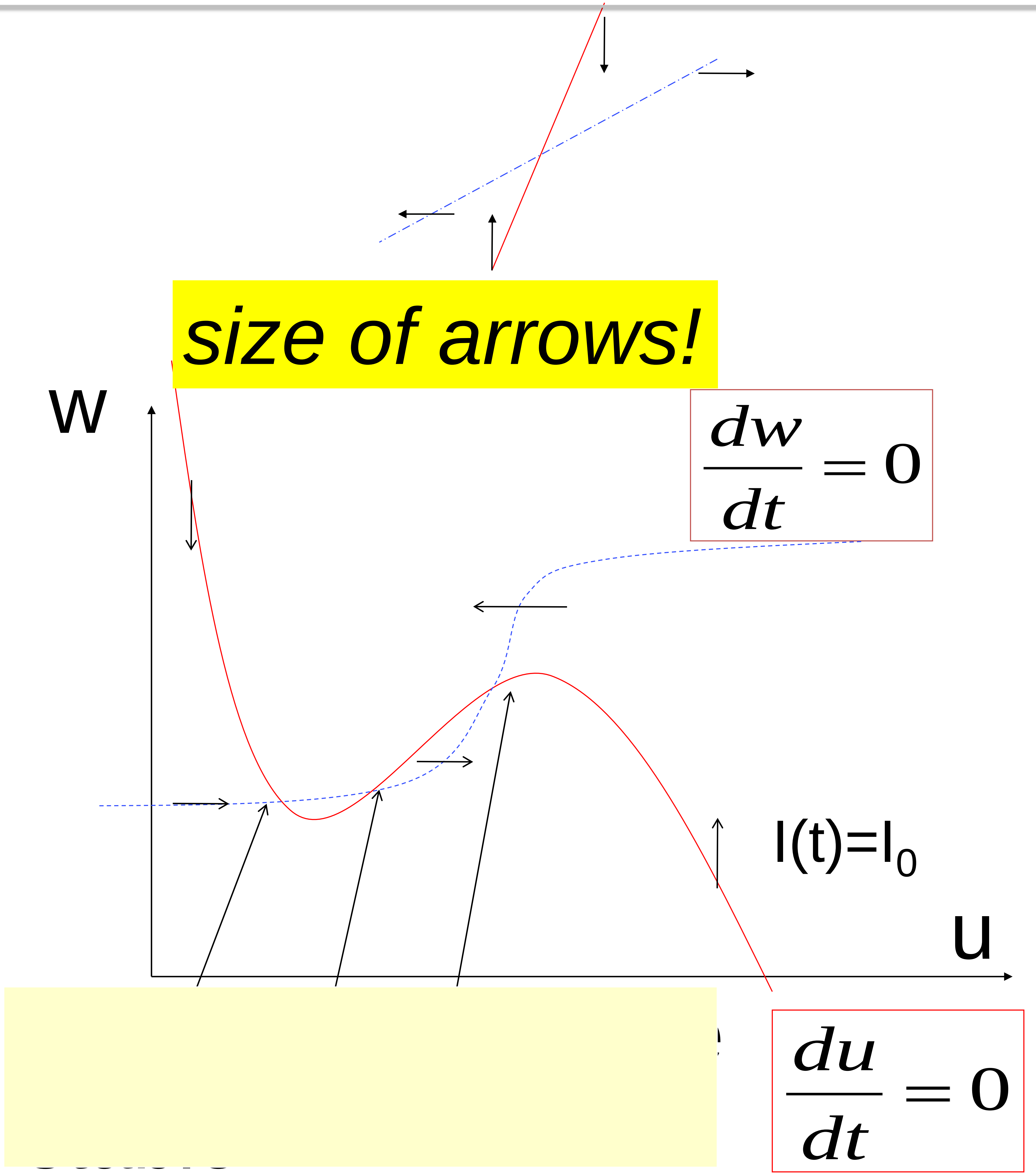
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

apply constant stimulus I_0

Saddle-node bifurcation

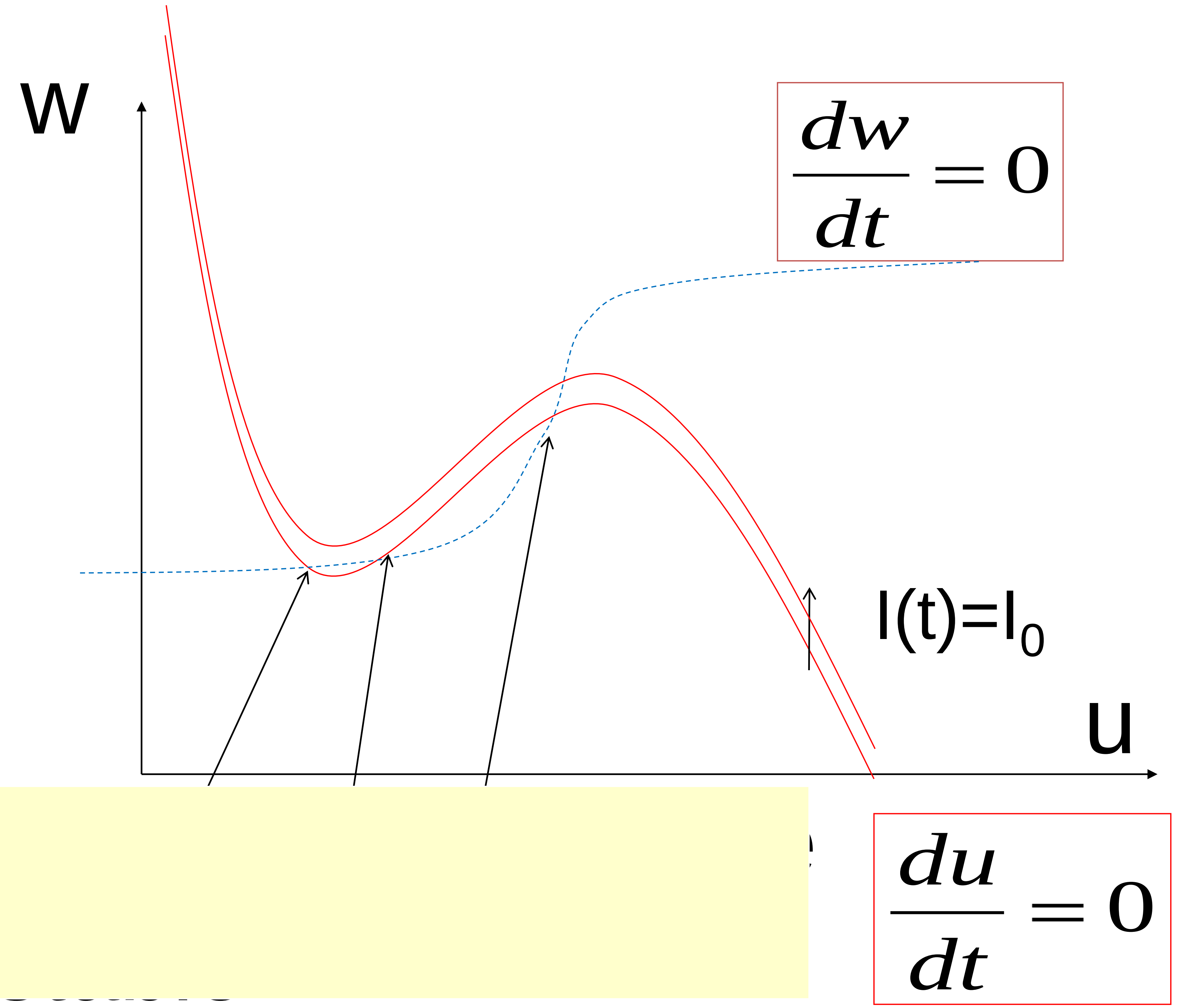


Saddle-node bifurcation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$



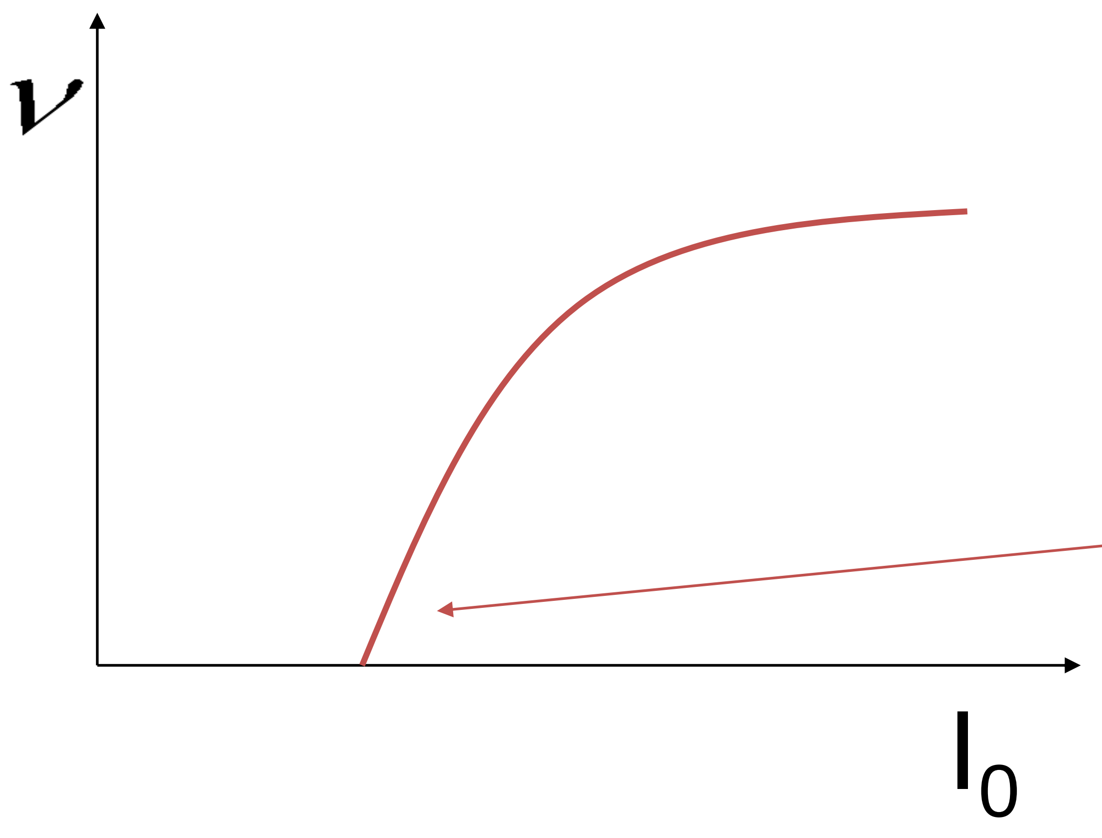
Blackboard:
- flow arrows,
- ghost/ruins

type I Model – constant input

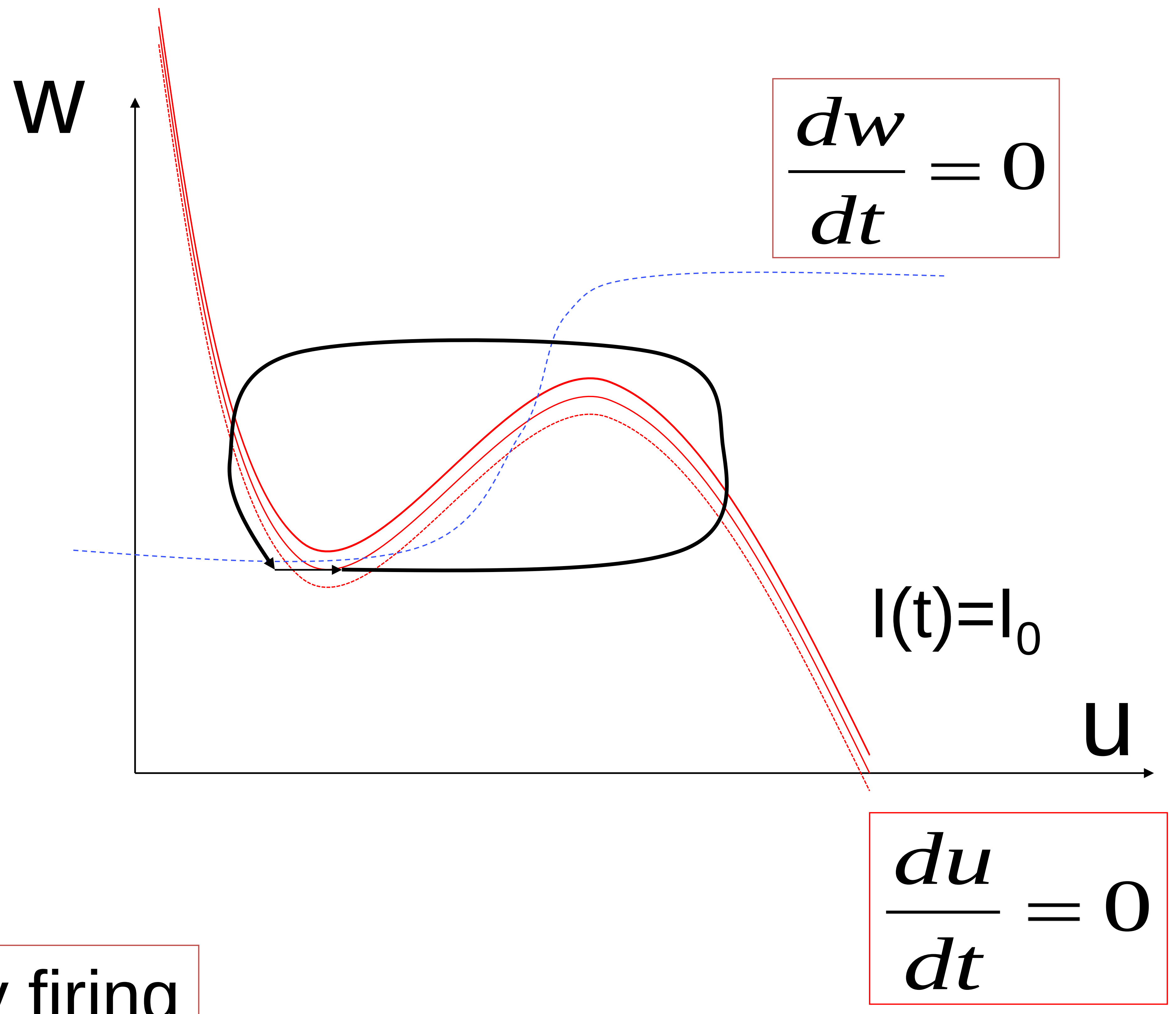
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$



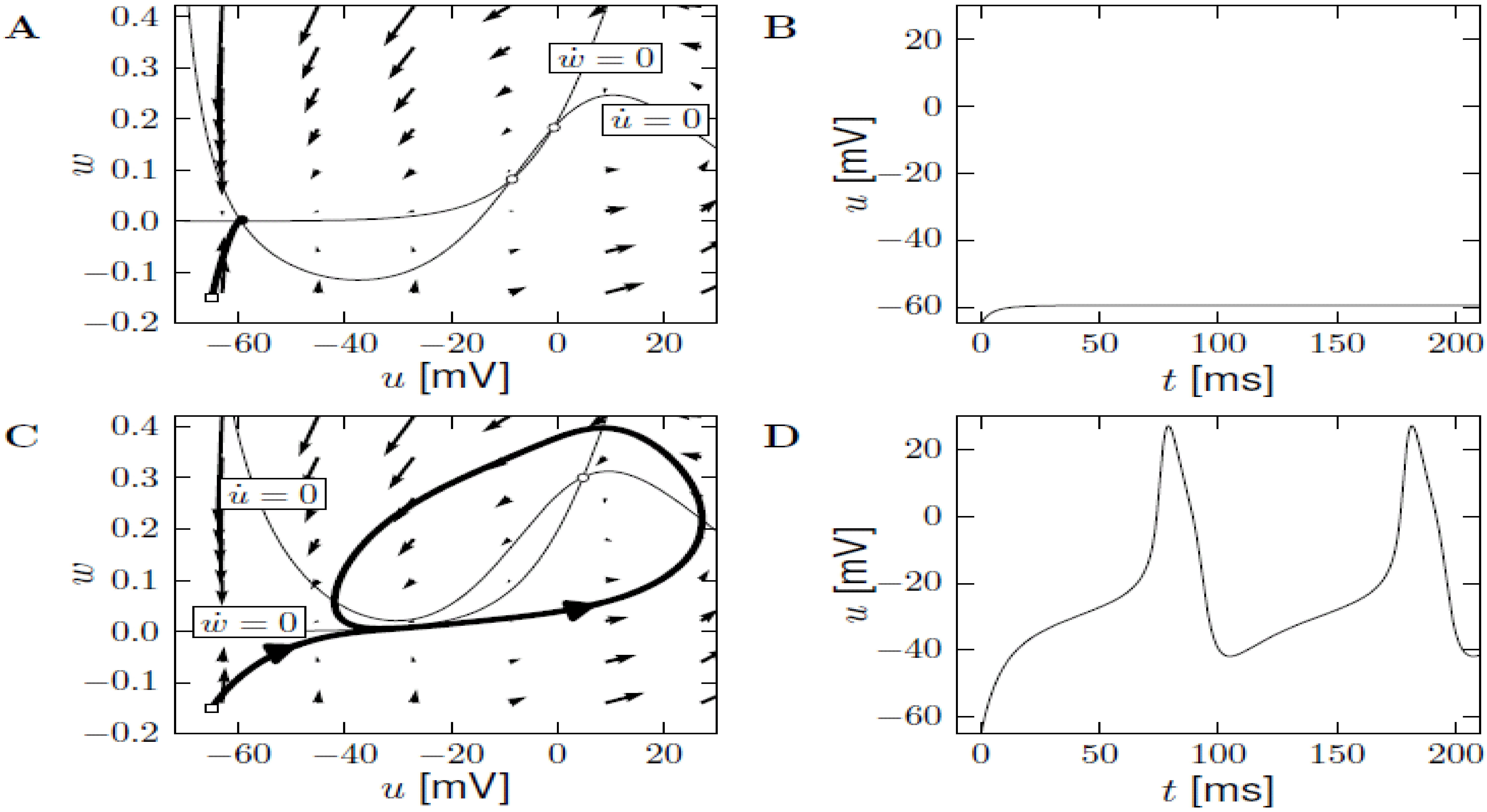
Low-frequency firing



Morris-Lecar, type I Model – constant input

I=0

I>I_c



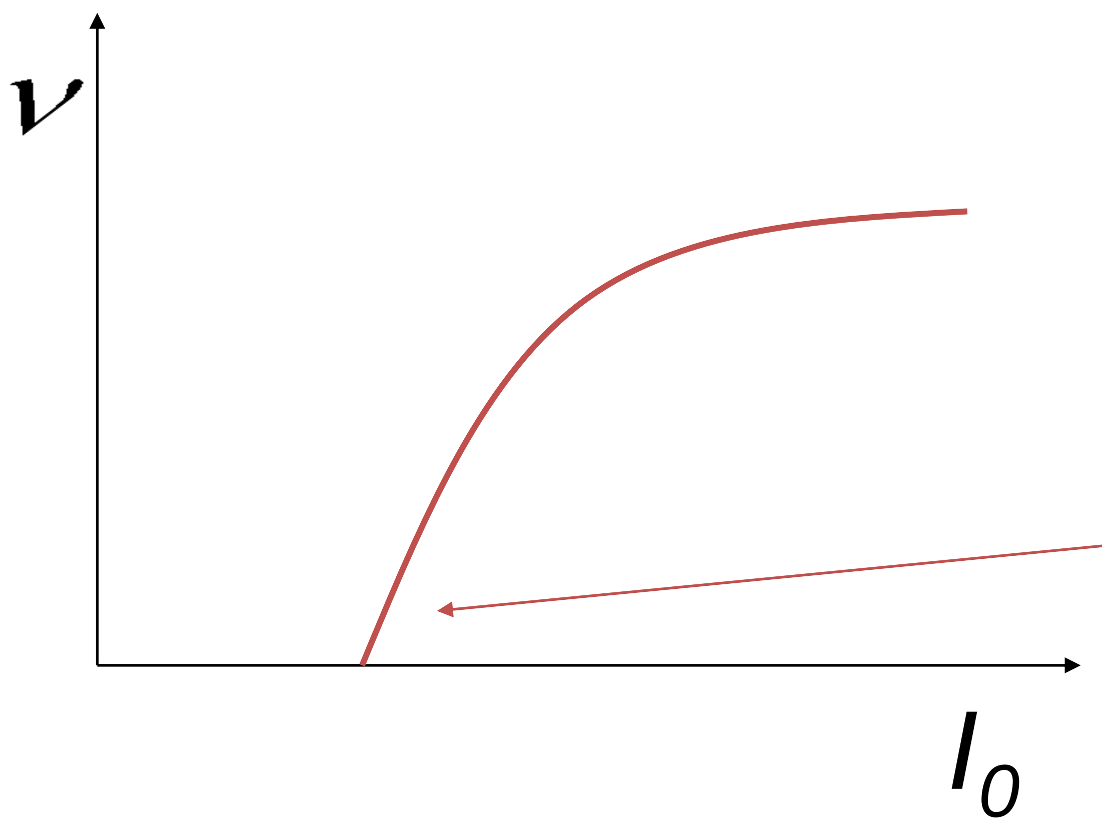
type I Model – Morris-Lecar: constant input

stimulus

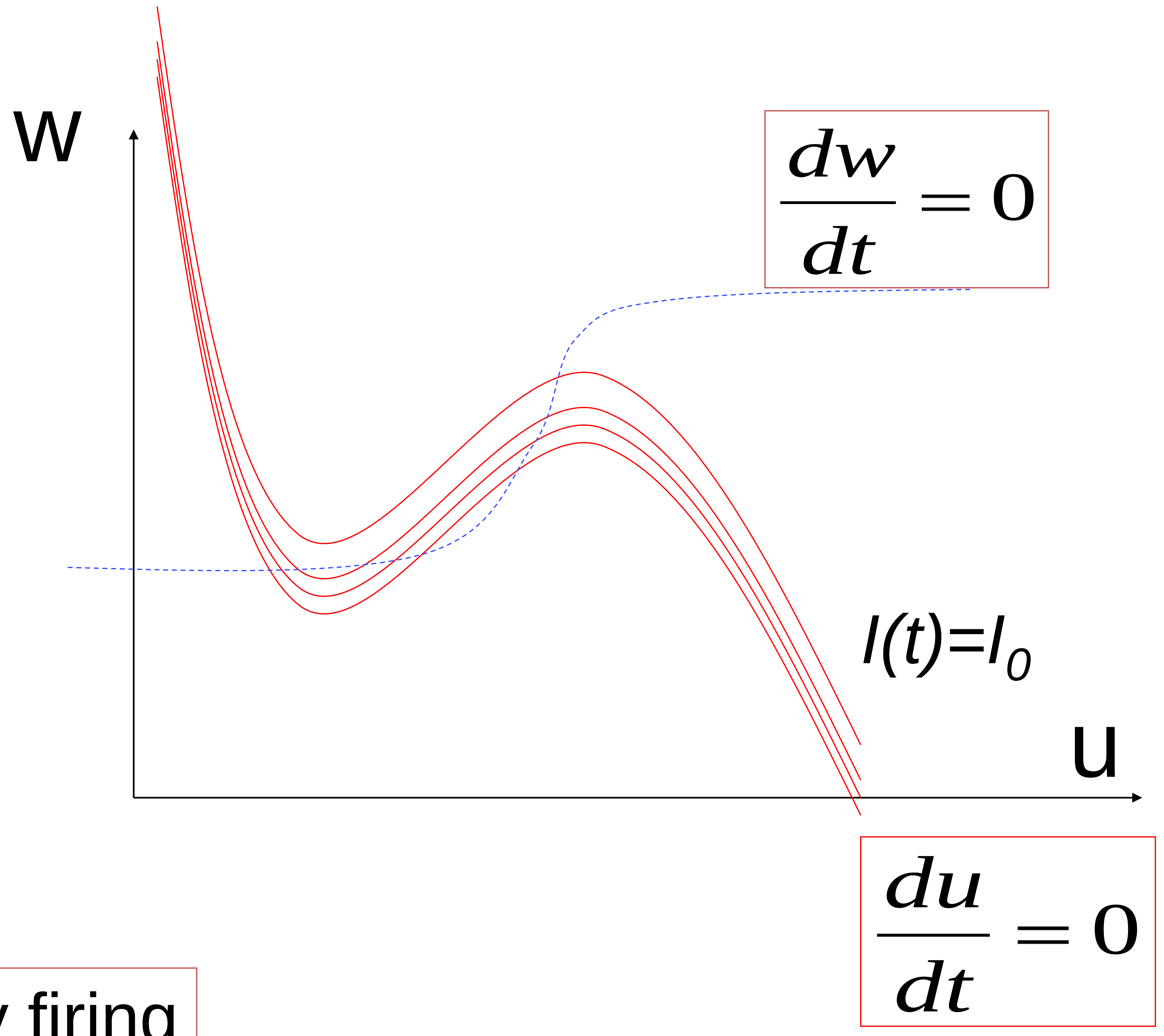
$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_{eff}(u)}$$

$$w_0(u) = 0.5[1 + \tanh(\tfrac{u-\theta}{d})]$$



Low-frequency firing

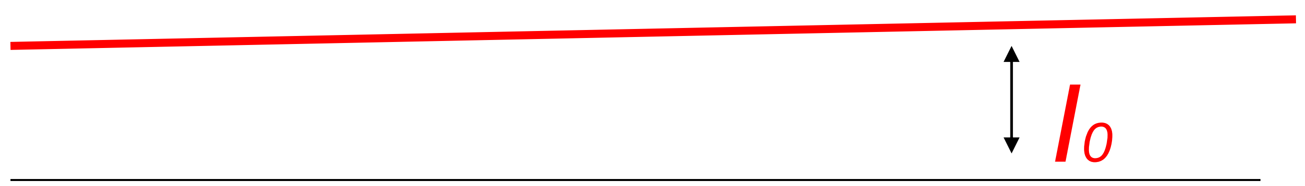


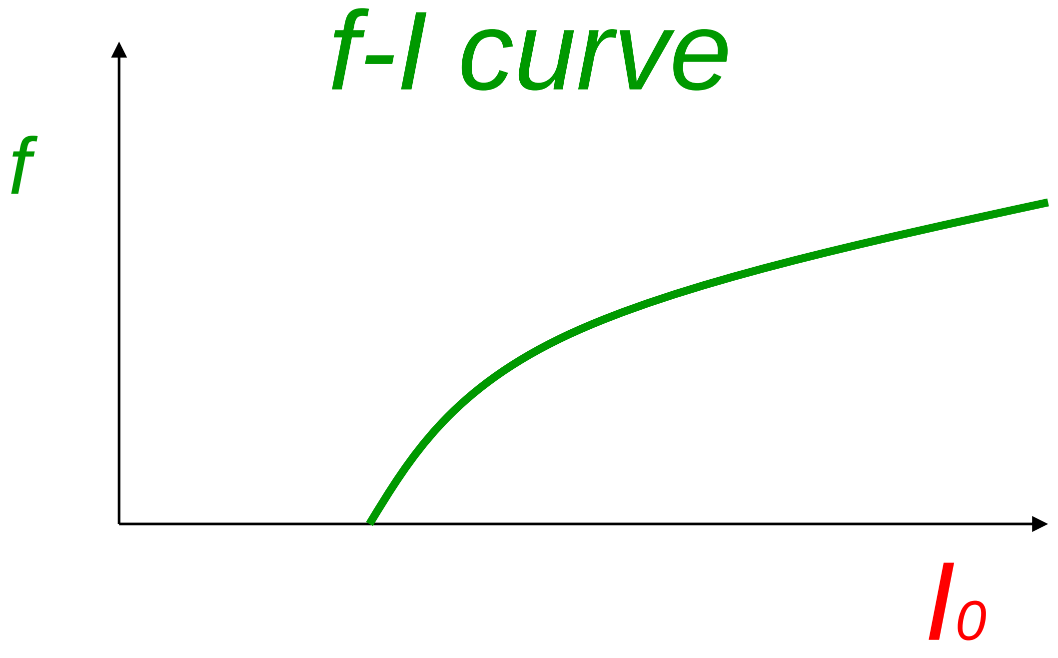
Type I and type II models

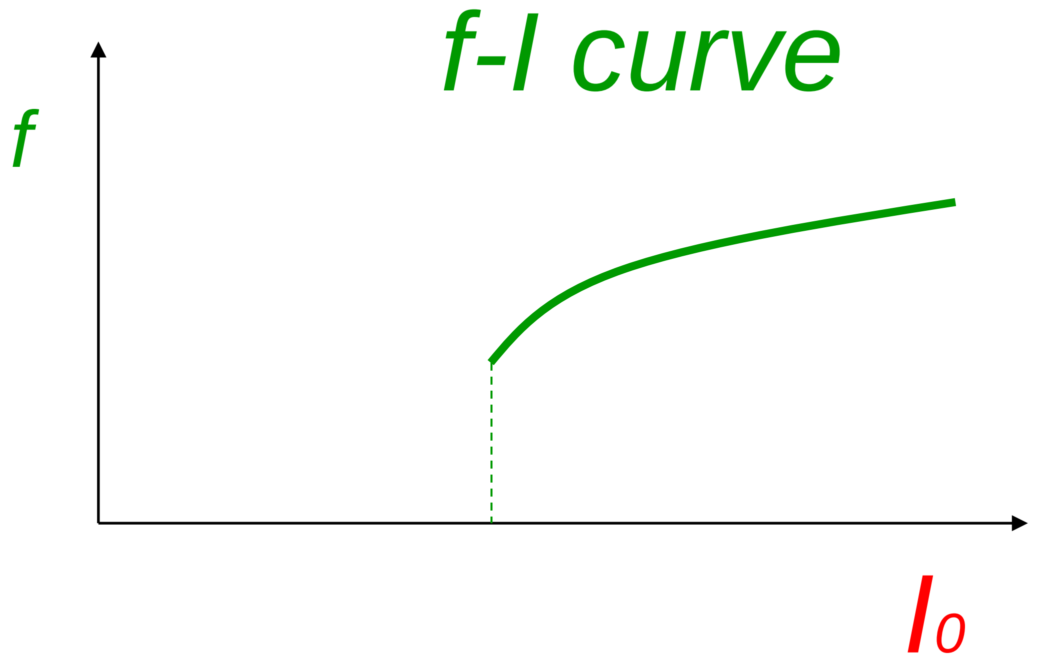
Response at firing threshold?

Type I	type II
Saddle-Node Onto limit cycle	For example: Subcritical Hopf

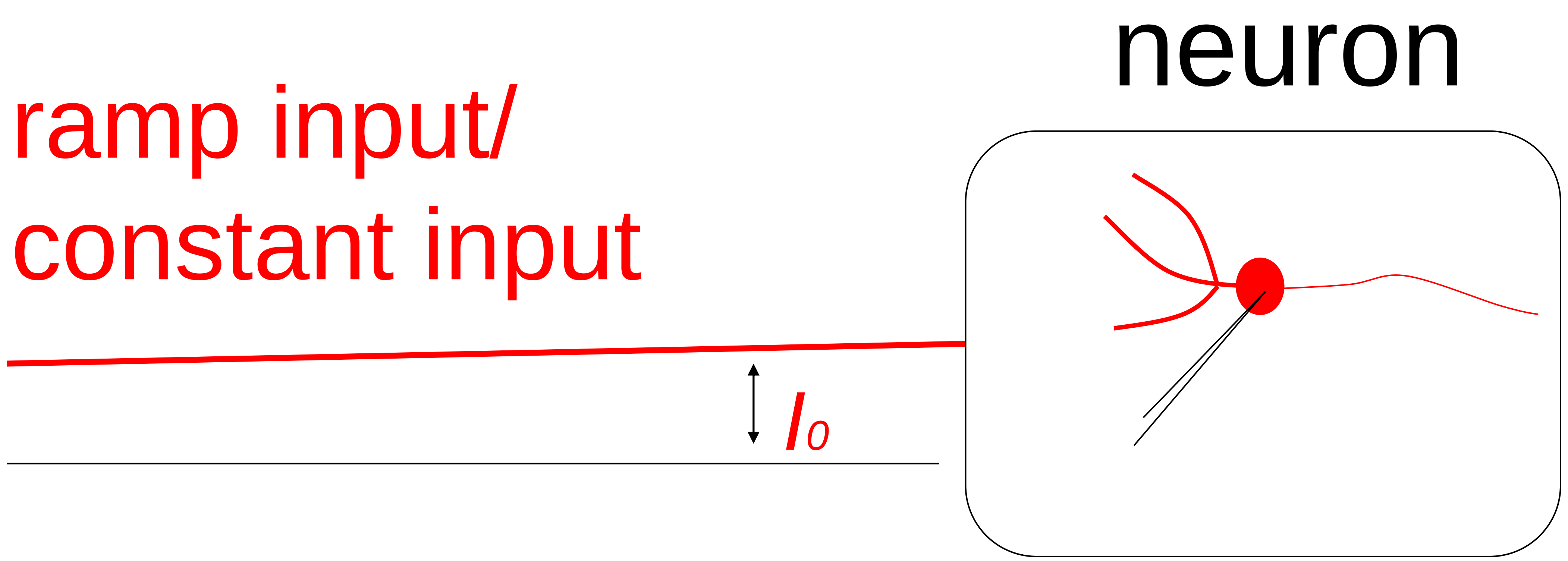
ramp input/
constant input



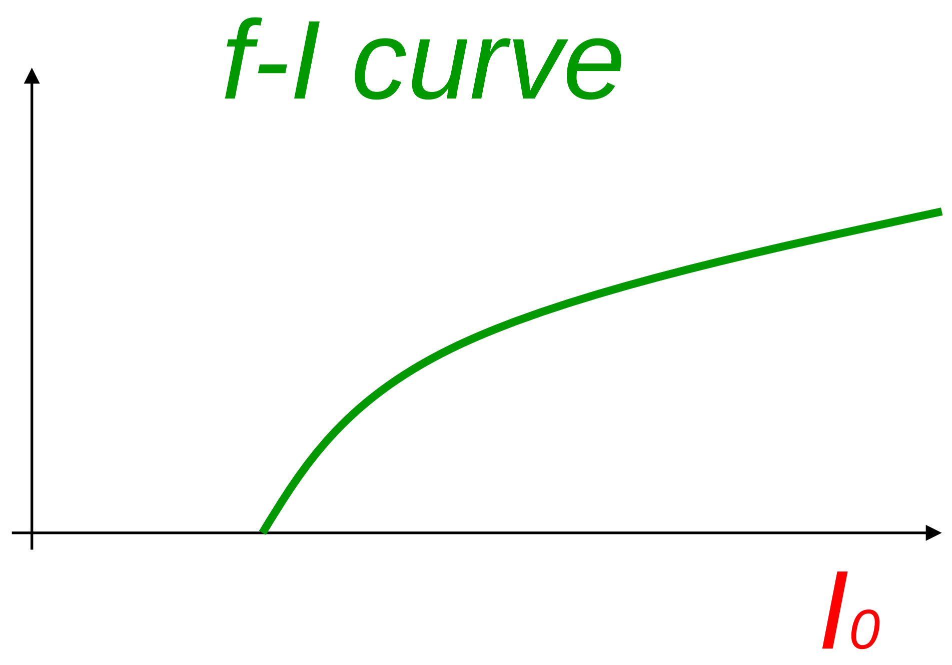


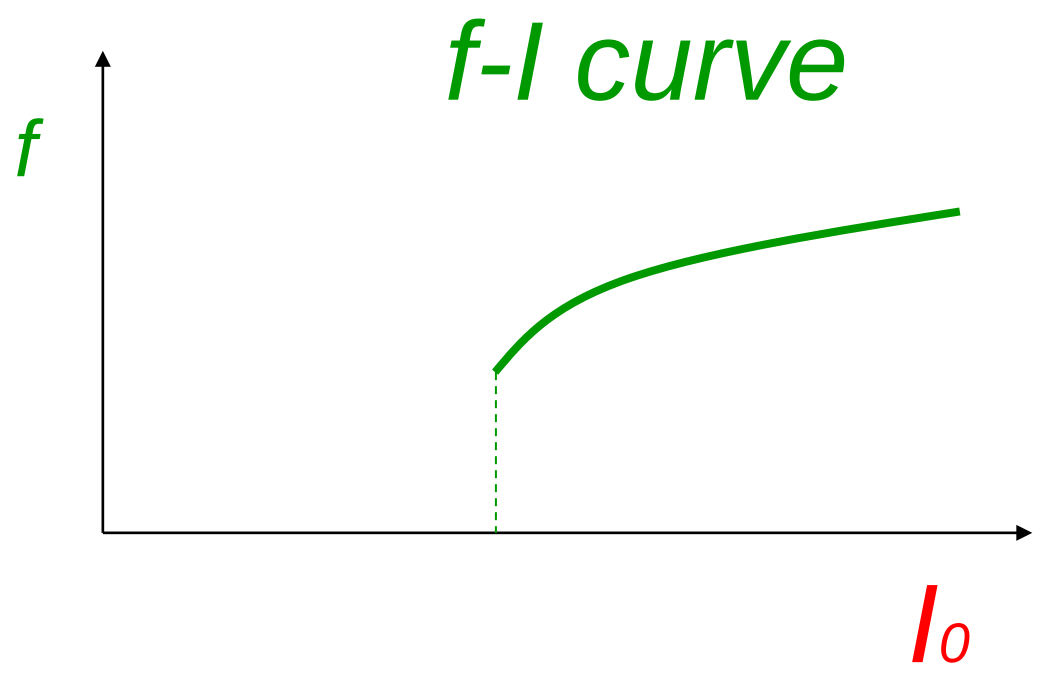


Neuronal Dynamics – 4.1. Type I and II Neuron Models



Type I and type II models





Neuronal Dynamics – Quiz 4.1.

A. 2-dimensional neuron model with (supercritical) saddle-node-onto-limit cycle bifurcation

[ ] The neuron model is of type II, because there is a jump in the f-I curve
[ ] The neuron model is of type I, because the f-I curve is continuous
[ ] The neuron model is of type I, if the limit cycle passes through a regime where the flow is very slow.
[ ] in the regime below the saddle-node-onto-limit cycle bifurcation, the neuron is at rest or will converge to the resting state.

B. Threshold in a 2-dimensional neuron model with subcritical Hopf bifurcation

[ ] The neuron model is of type II, because there is a jump in the f-I curve
[ ] The neuron model is of type I, because the f-I curve is continuous
[ ] in the regime below the Hopf bifurcation, the neuron is at rest or will necessarily converge to the resting state

Week 4 – part 1: Reducing Detail – 2D models

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

Week 4

– Reducing detail

- Adding detail

Wulfram Gerstner

EPFL, Lausanne, Switzerland

3.1 **From Hodgkin-Huxley to 2D**

3.2 **Phase Plane Analysis**

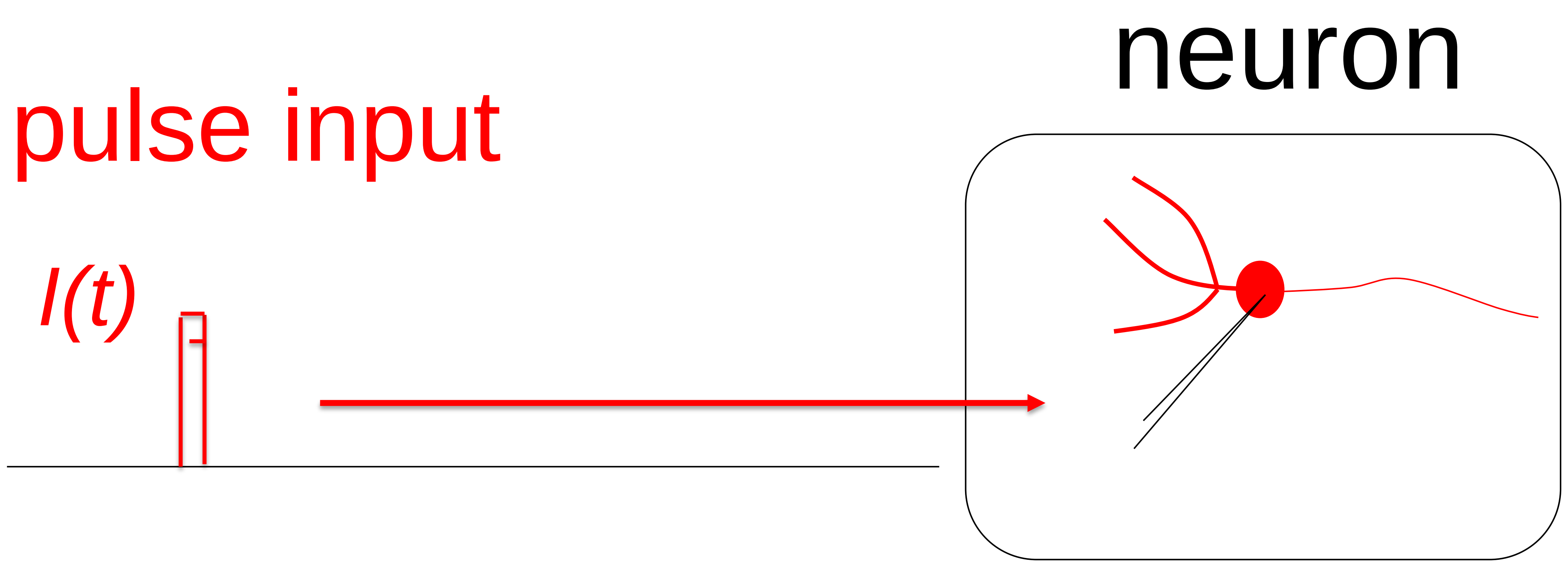
3.3 **Analysis of a 2D Neuron Model**

4.1 **Type I and II Neuron Models**

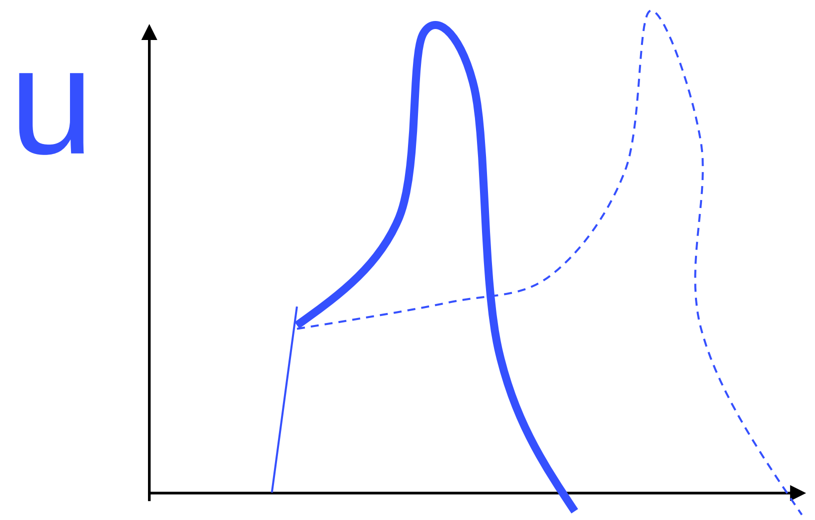
- limit cycles
- where is the firing threshold?
- separation of time scales

4.2. Adding detail

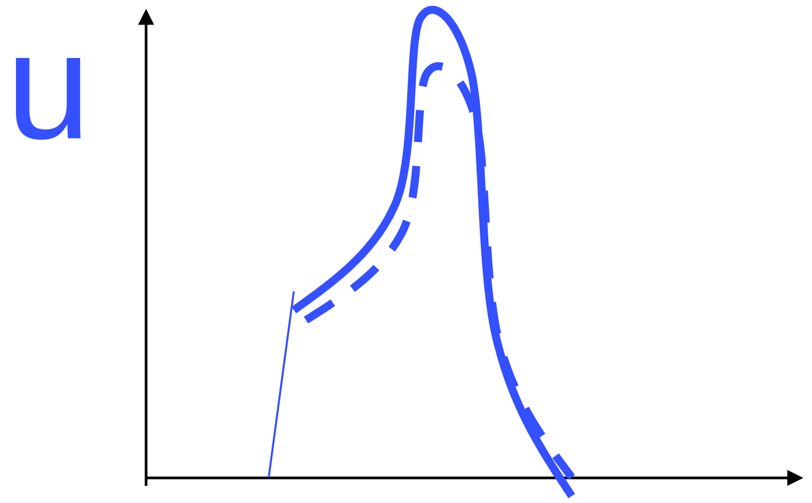
Neuronal Dynamics – 4.1. Threshold in 2dim. Neuron Models



Delayed spike



Reduced amplitude



Bifurcations in neural modeling,
Type I/II neuron models,
Canonical simplified models

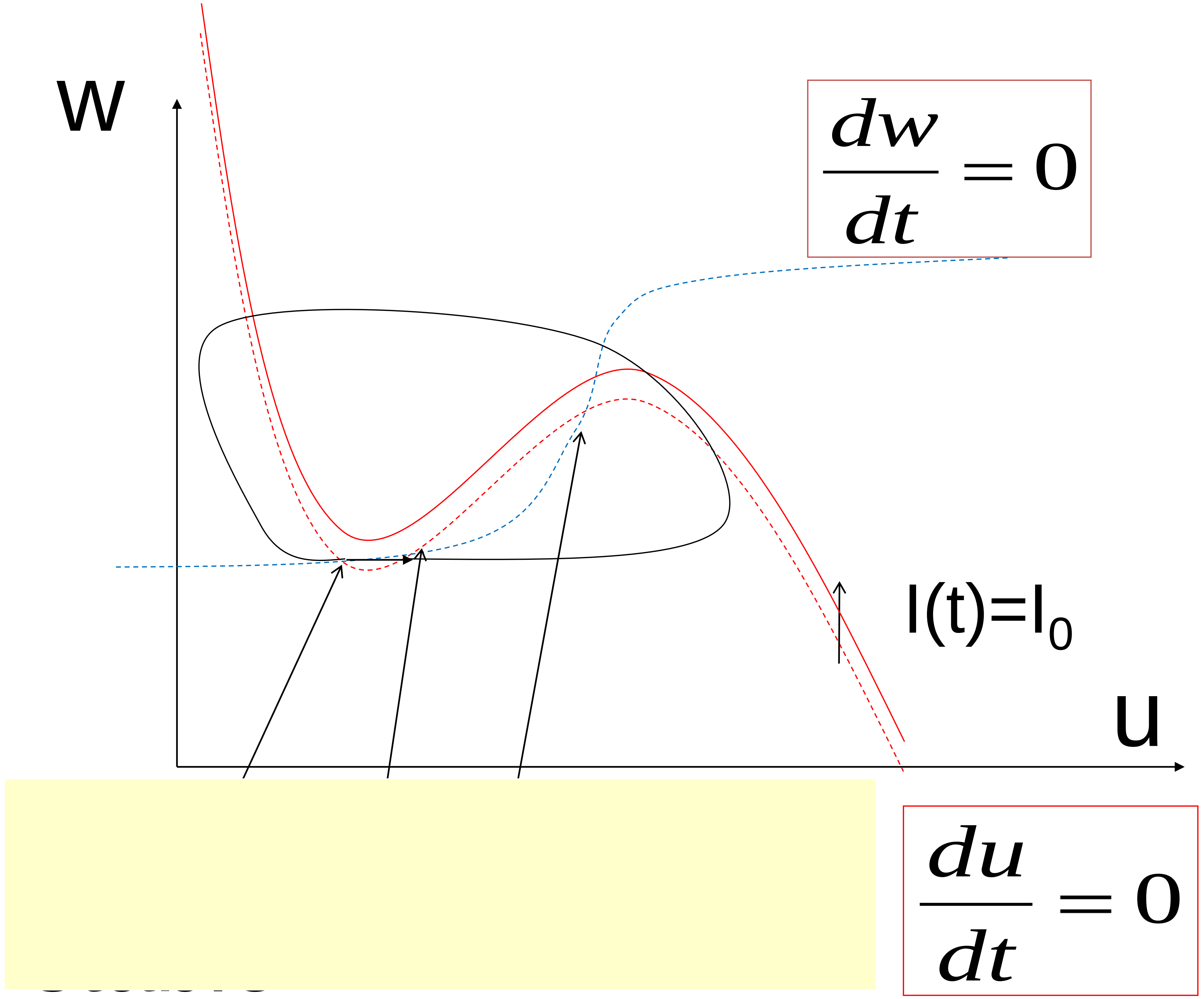
Nancy Koppell,
Bart Ermentrout,
John Rinzel,
Eugene Izhikevich
and many others

Review: Saddle-node onto limit cycle bifurcation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$



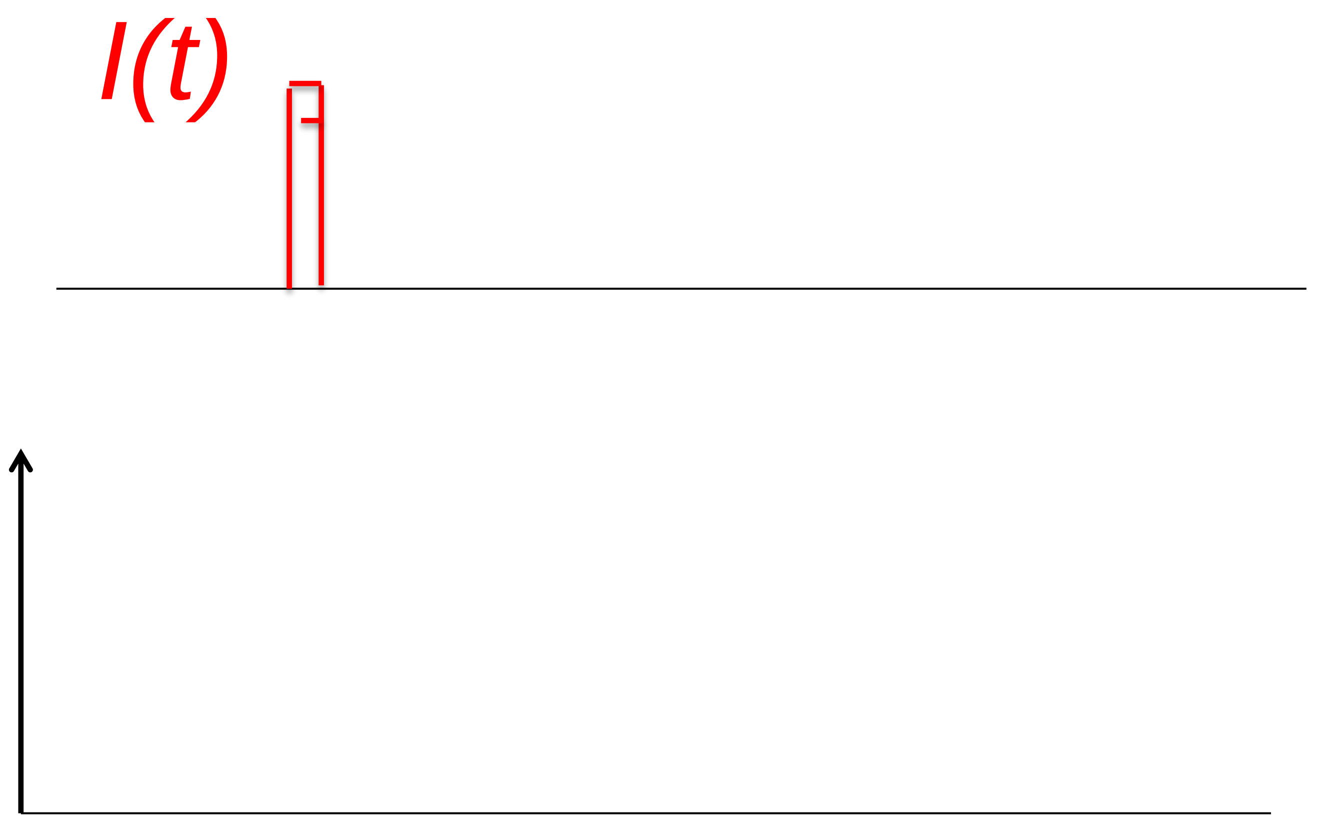
Neuronal Dynamics – 4.1 Pulse input

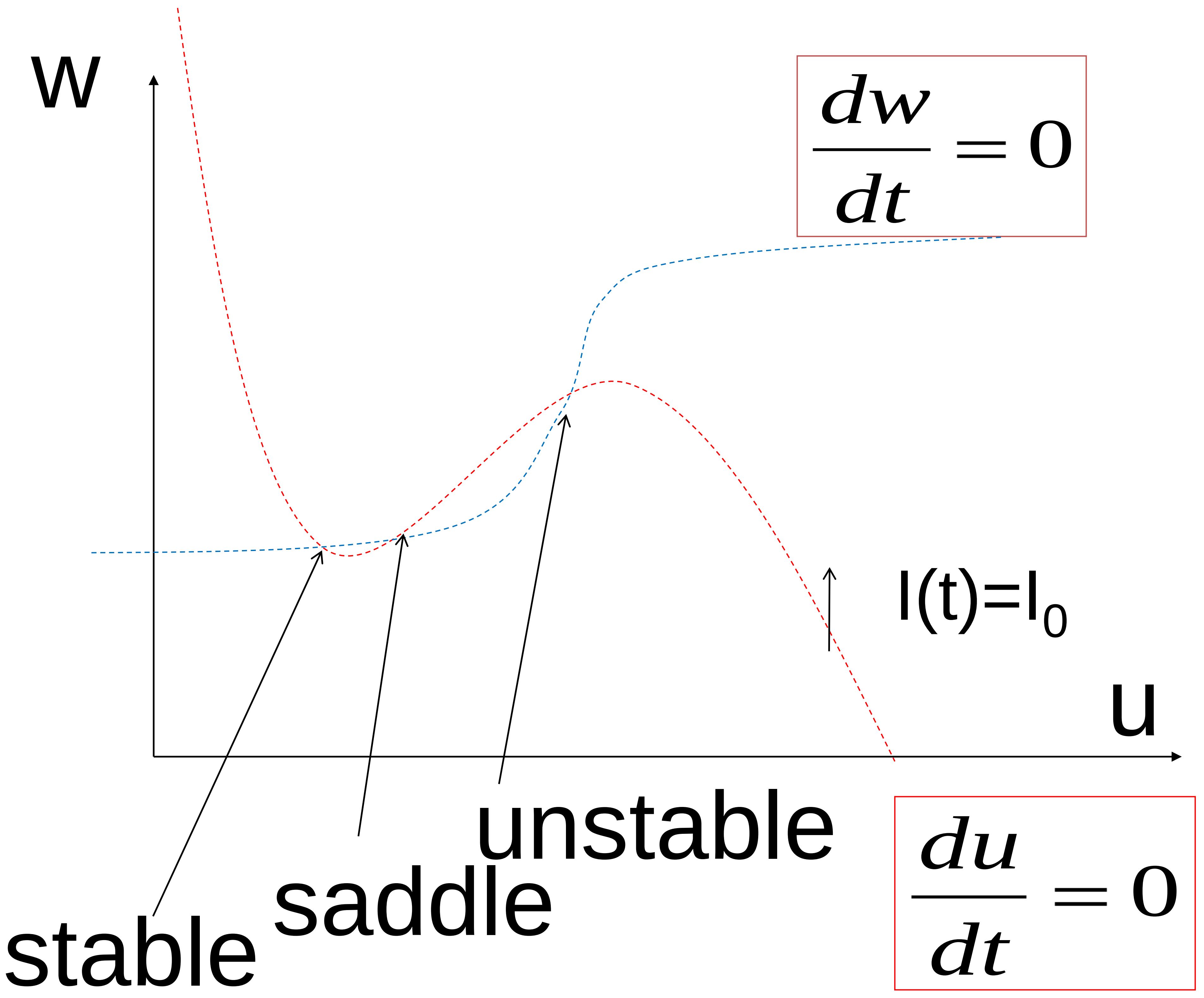
stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input





4.1 Type I model: Pulse input

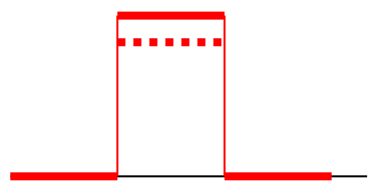
stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

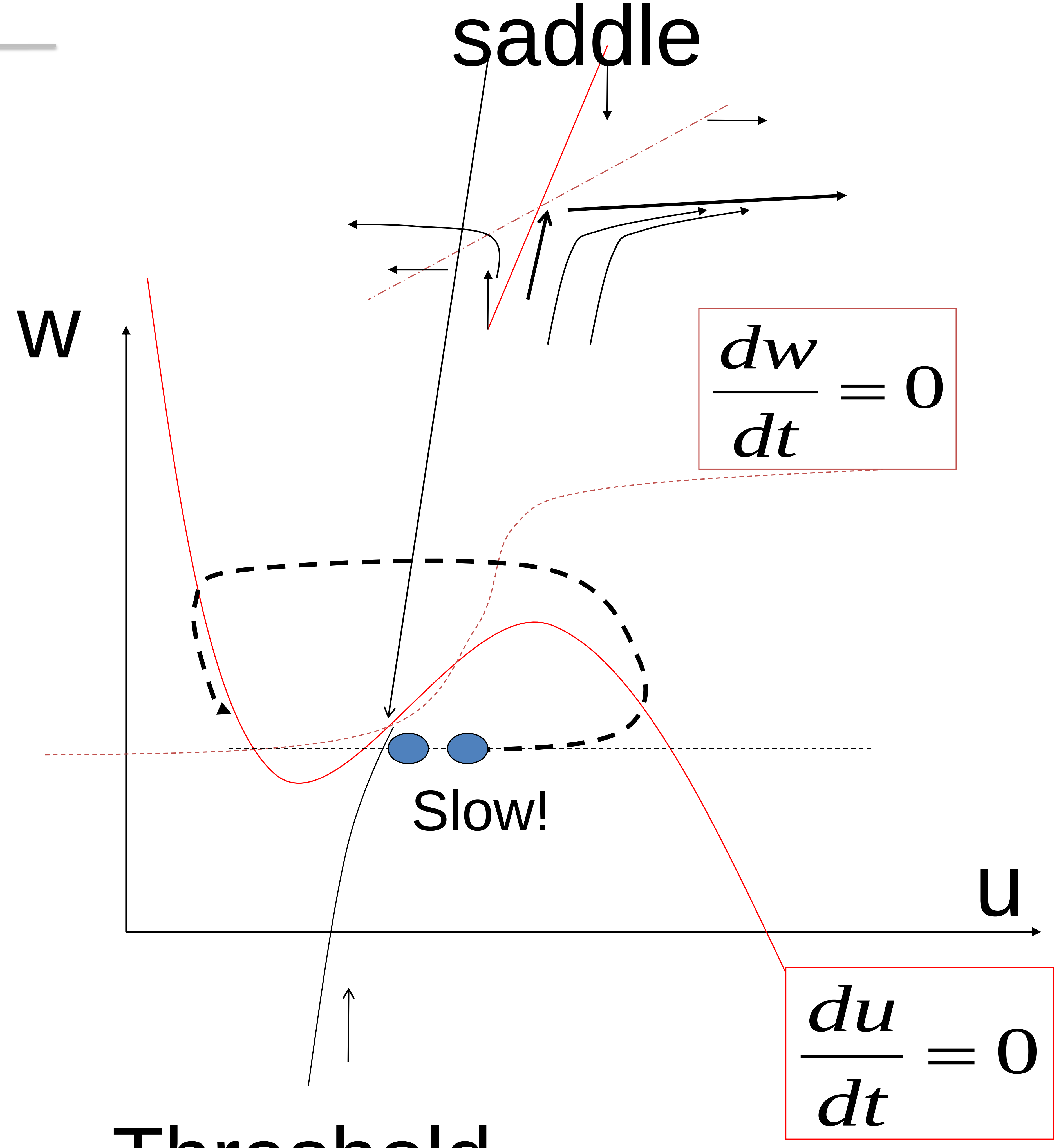
$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input

I(t)

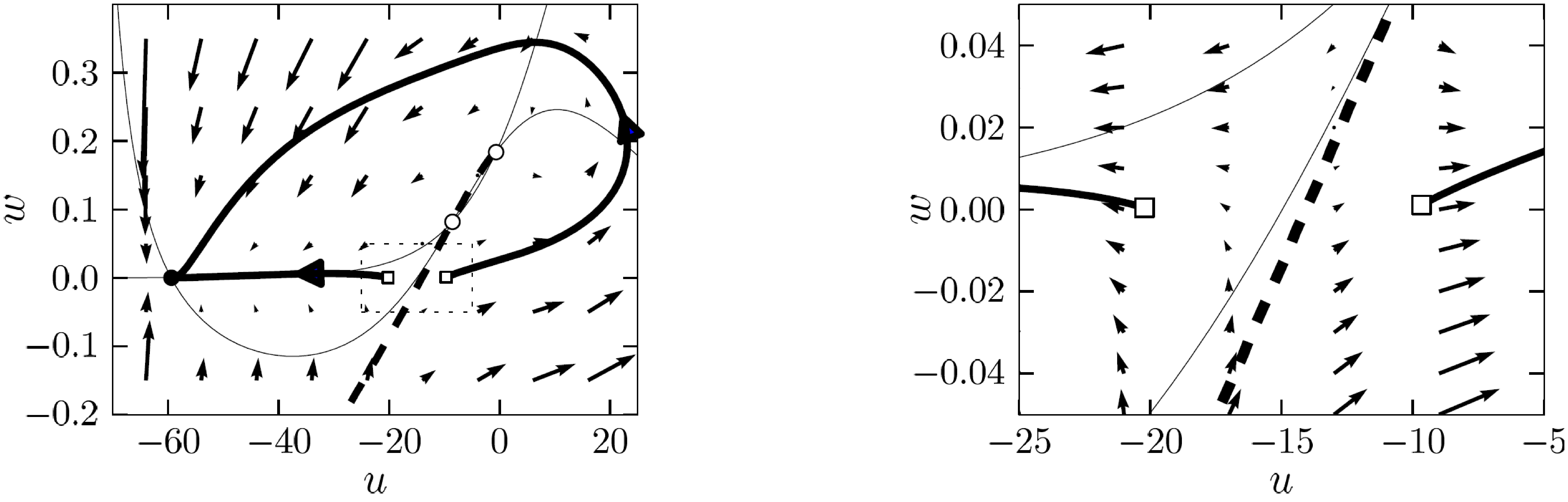


blackboard



Threshold
for pulse input

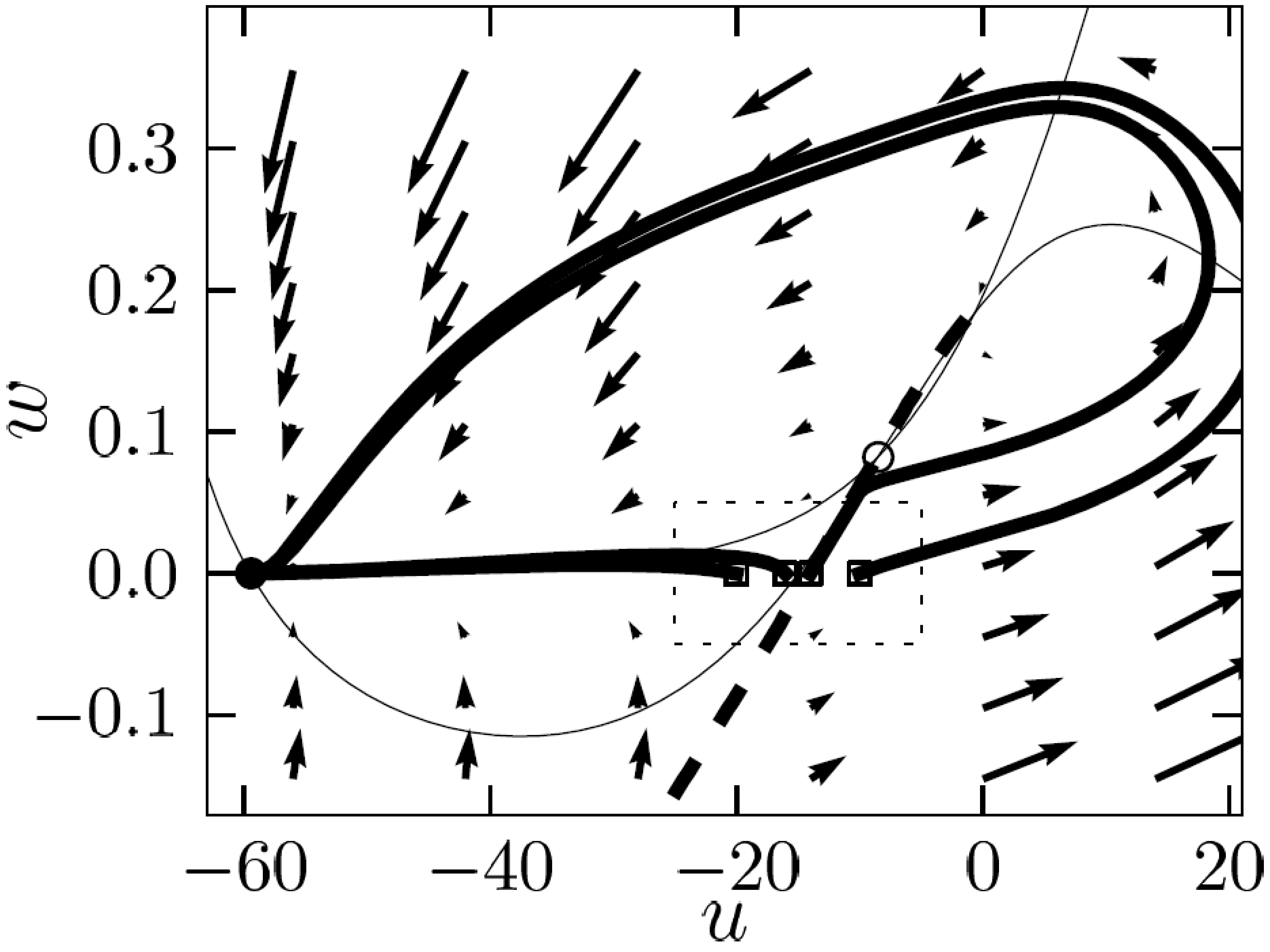
4.1 Type I model: Threshold for Pulse input

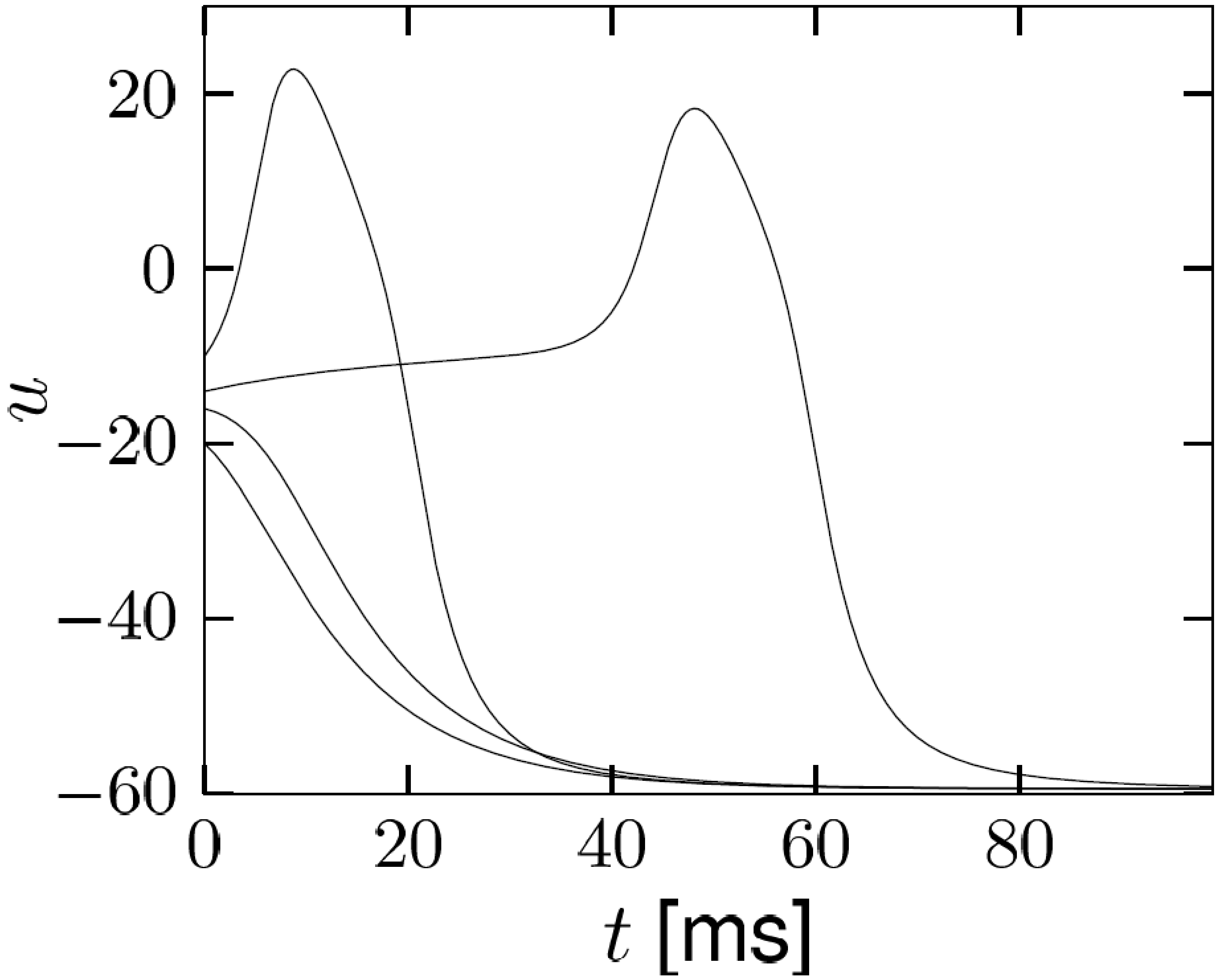


Stable manifold plays role of 'Threshold' (for pulse input)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

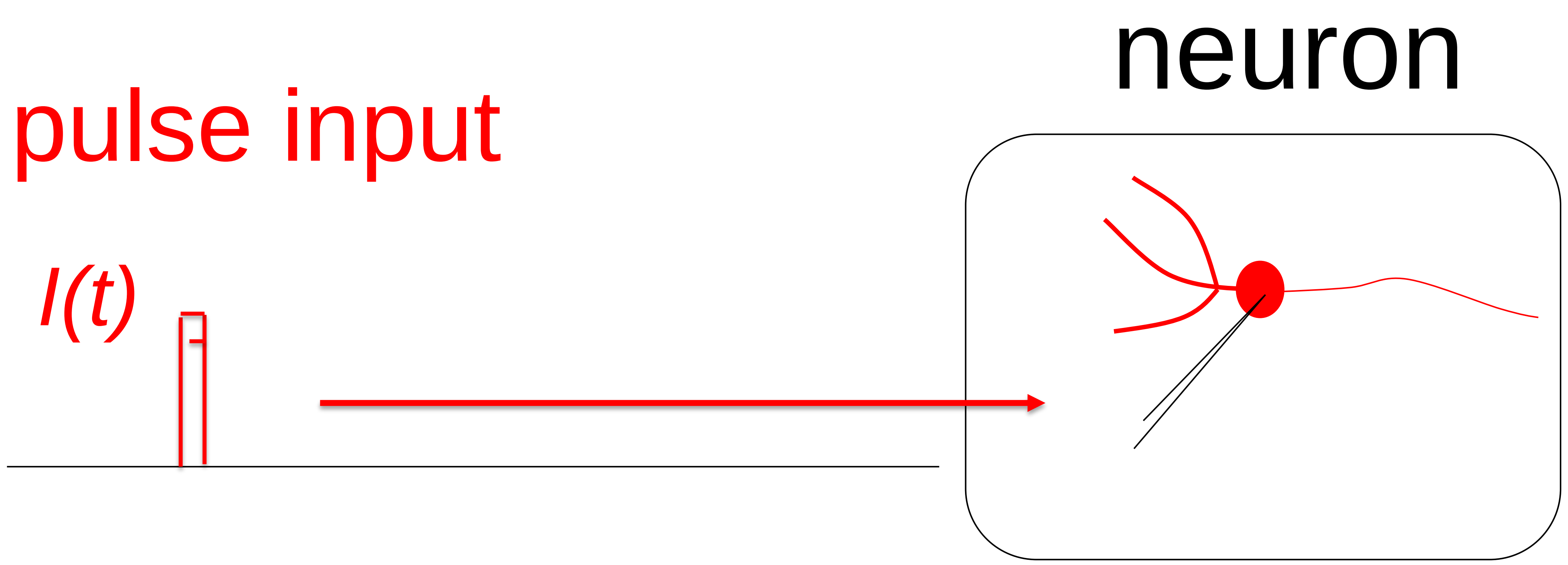
4.1 Type I model: Delayed spike initation for Pulse input



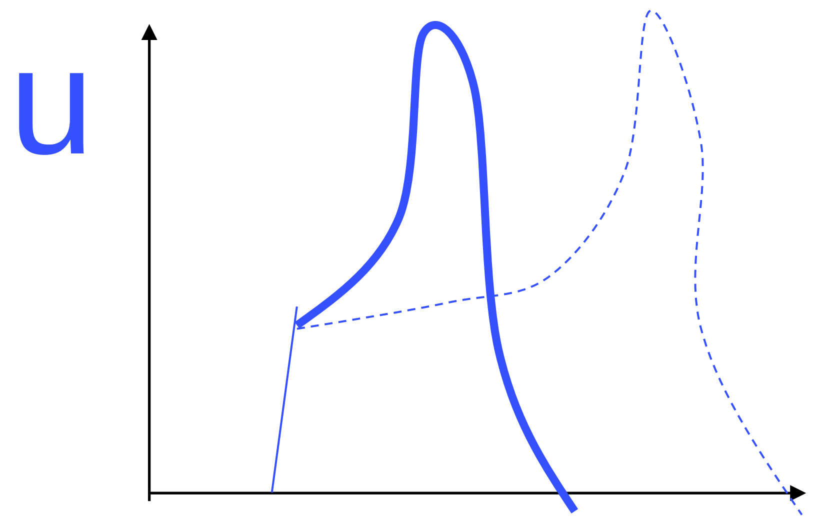


Delayed spike initiation close to 'Threshold' (for pulse input)

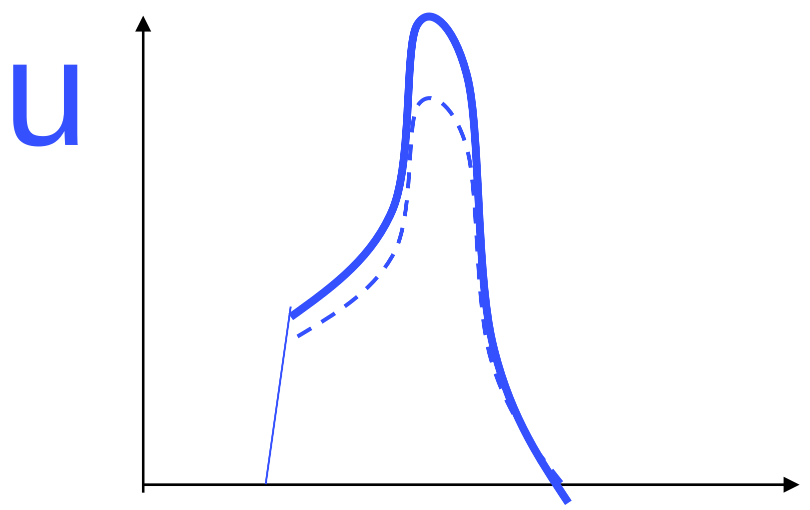
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)



Delayed spike



Reduced amplitude



NOW: model with subc. Hopf

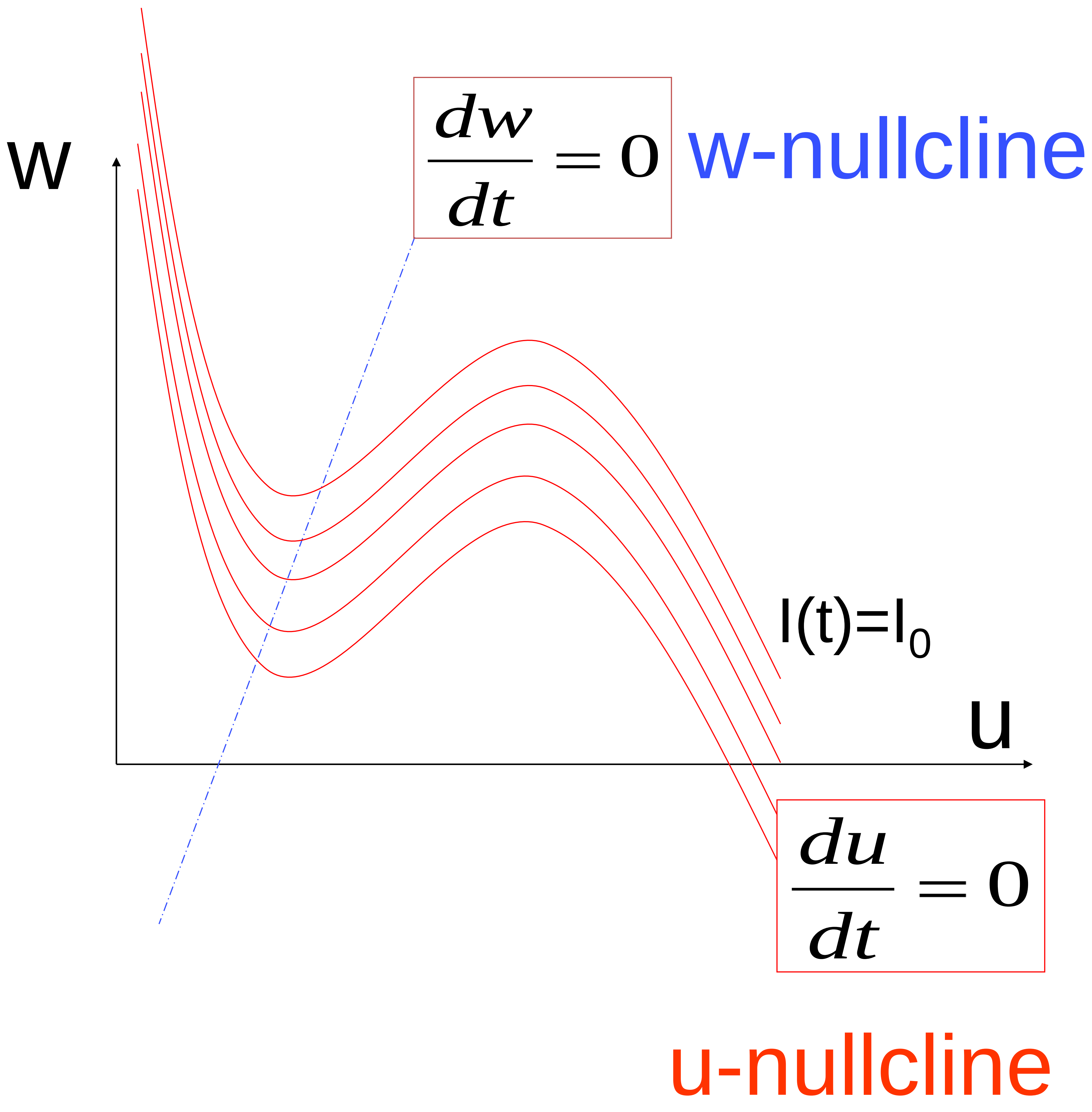
Review: FitzHugh-Nagumo Model: Hopf bifurcation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

apply constant stimulus I_0



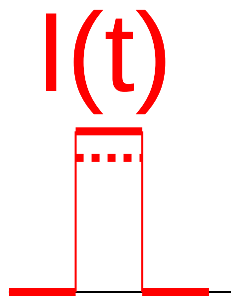
FitzHugh-Nagumo Model - pulse input

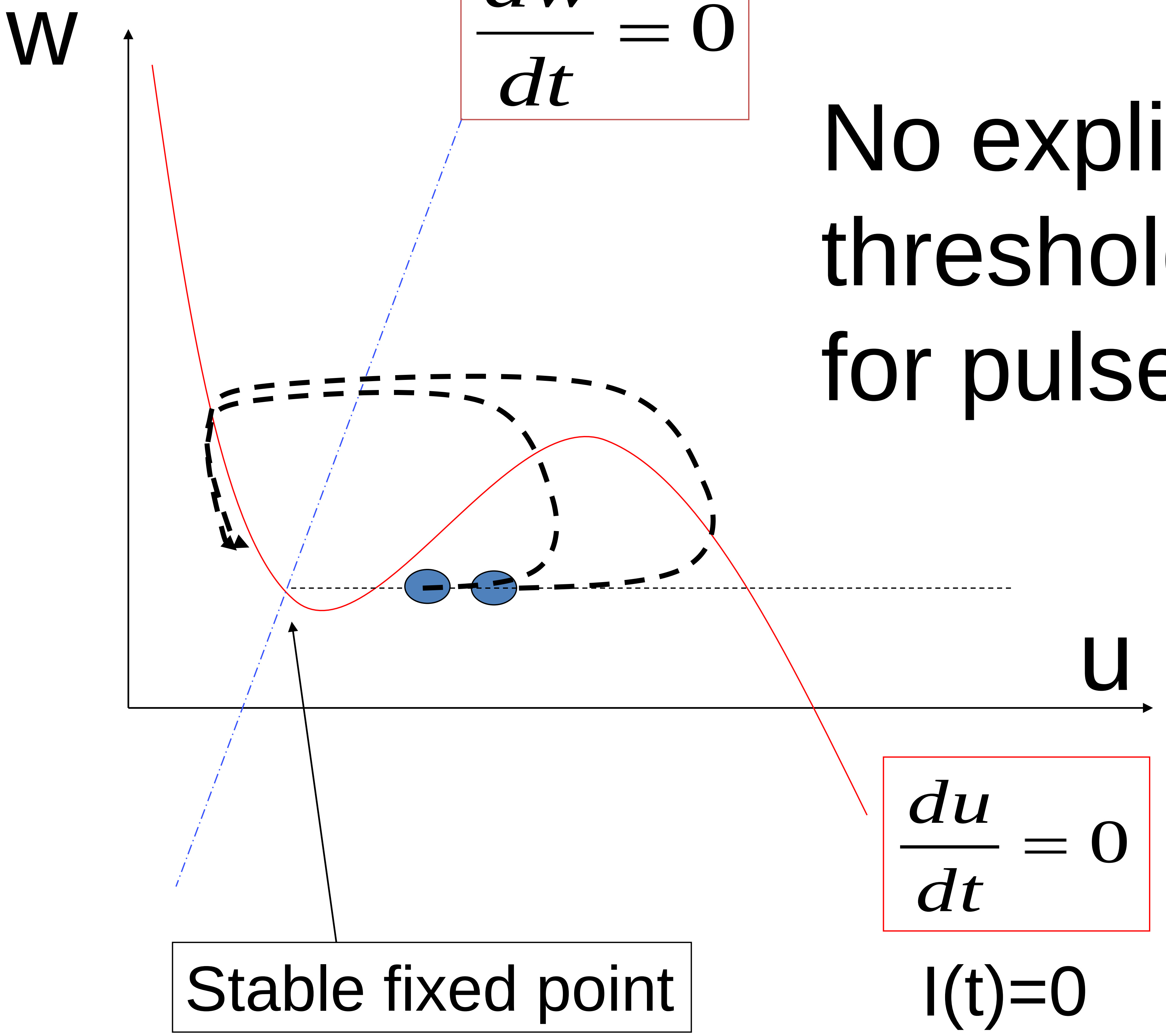
stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input





$$\frac{dw}{dt} = 0$$

$$\frac{du}{dt} = 0$$

No explicit threshold for pulse input

Week 4 – part 1: Reducing Detail – 2D models

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

Week 4

– Reducing detail

- Adding detail

Wulfram Gerstner

EPFL, Lausanne, Switzerland

3.1 **From Hodgkin-Huxley to 2D**

3.2 **Phase Plane Analysis**

3.3 **Analysis of a 2D Neuron Model**

4.1 **Type I and II Neuron Models**

- limit cycles
- where is the firing threshold?
- separation of time scales

4.2. Dendrites

FitzHugh-Nagumo Model - pulse input threshold?

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

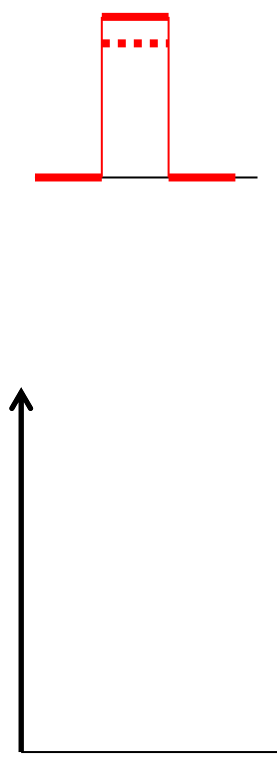
$$\tau_w \frac{dw}{dt} = G(u,w)$$

Separation of time scales

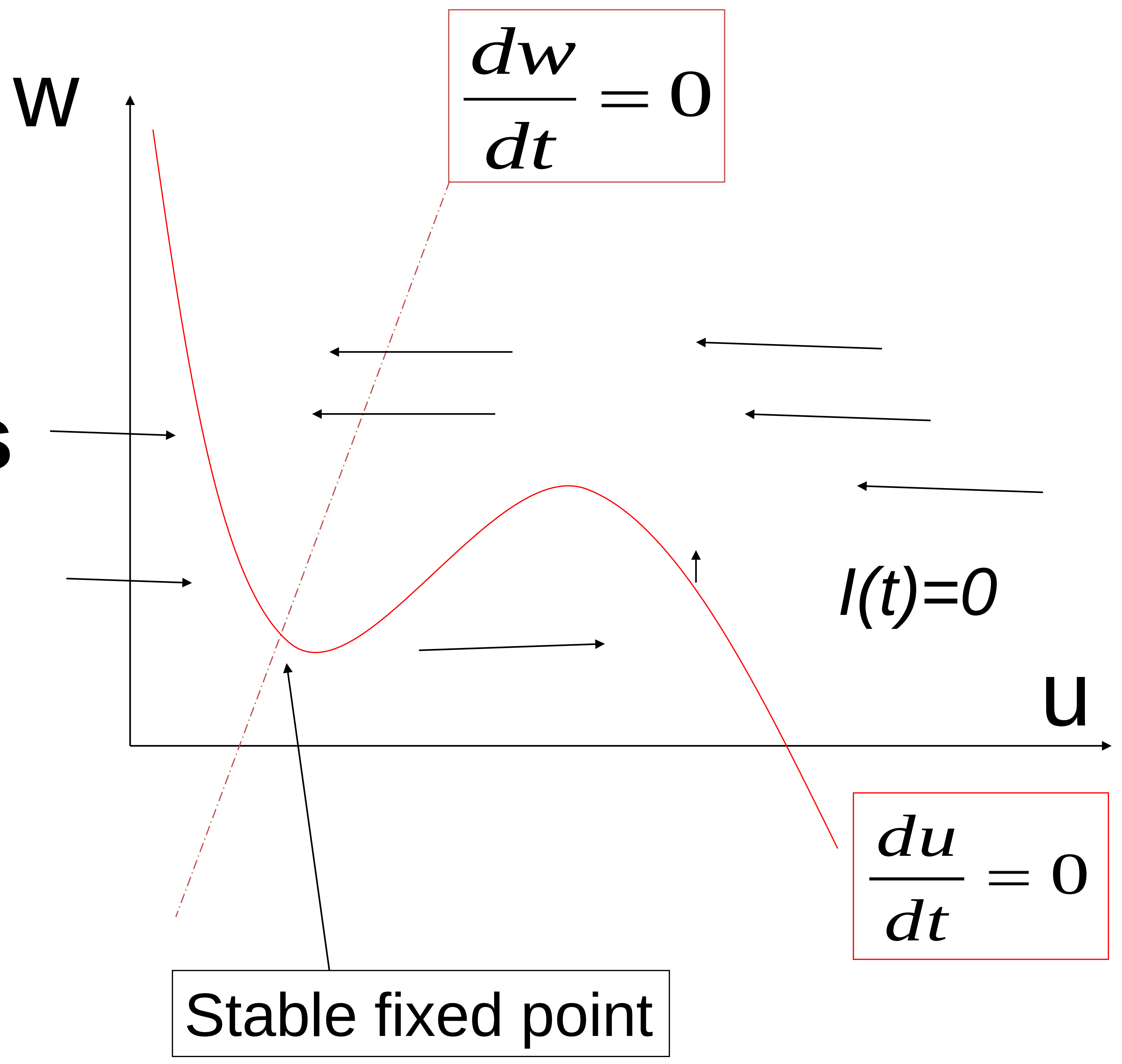
pulse input

$$\tau_w >> \tau_u$$

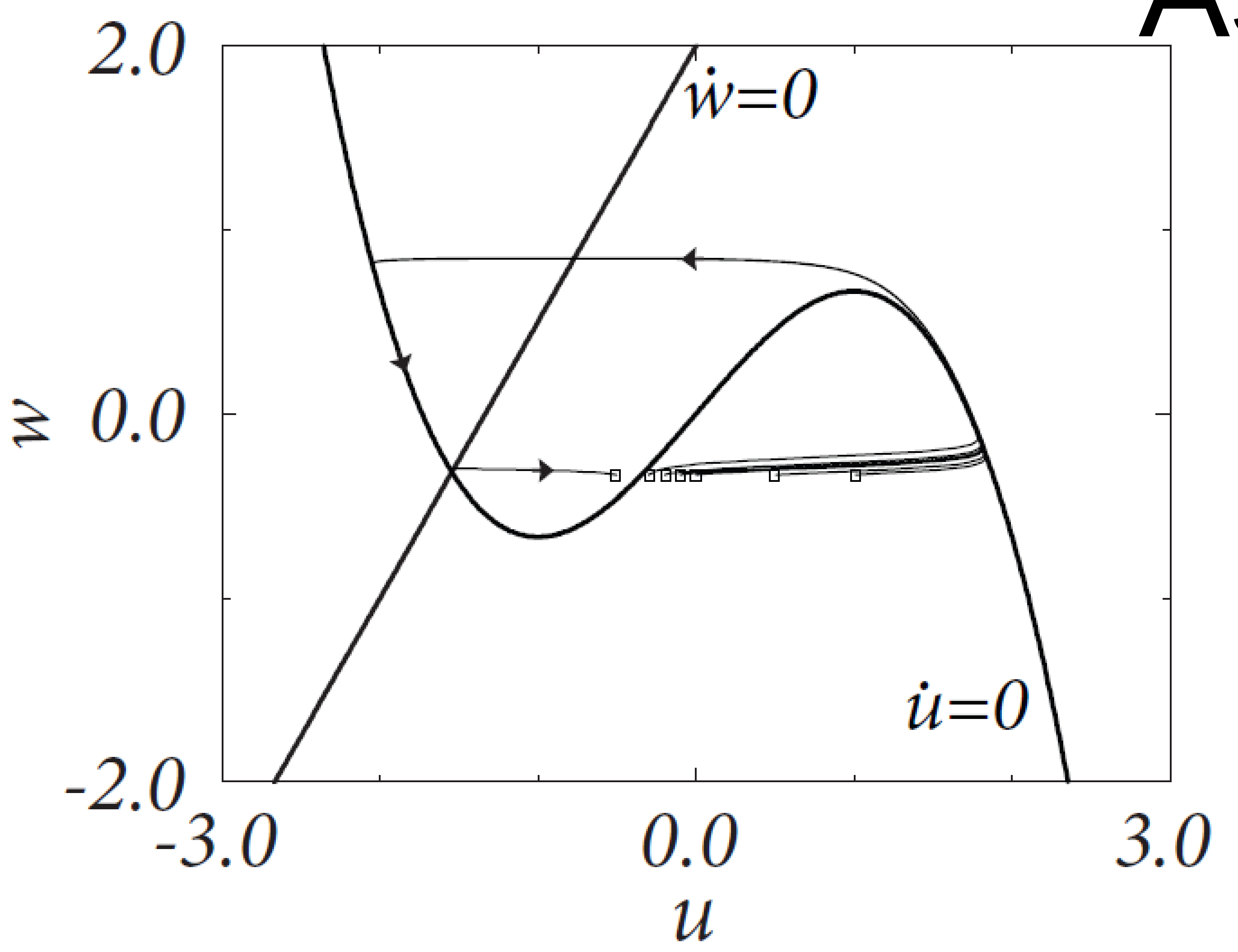
I(t)



blackboard

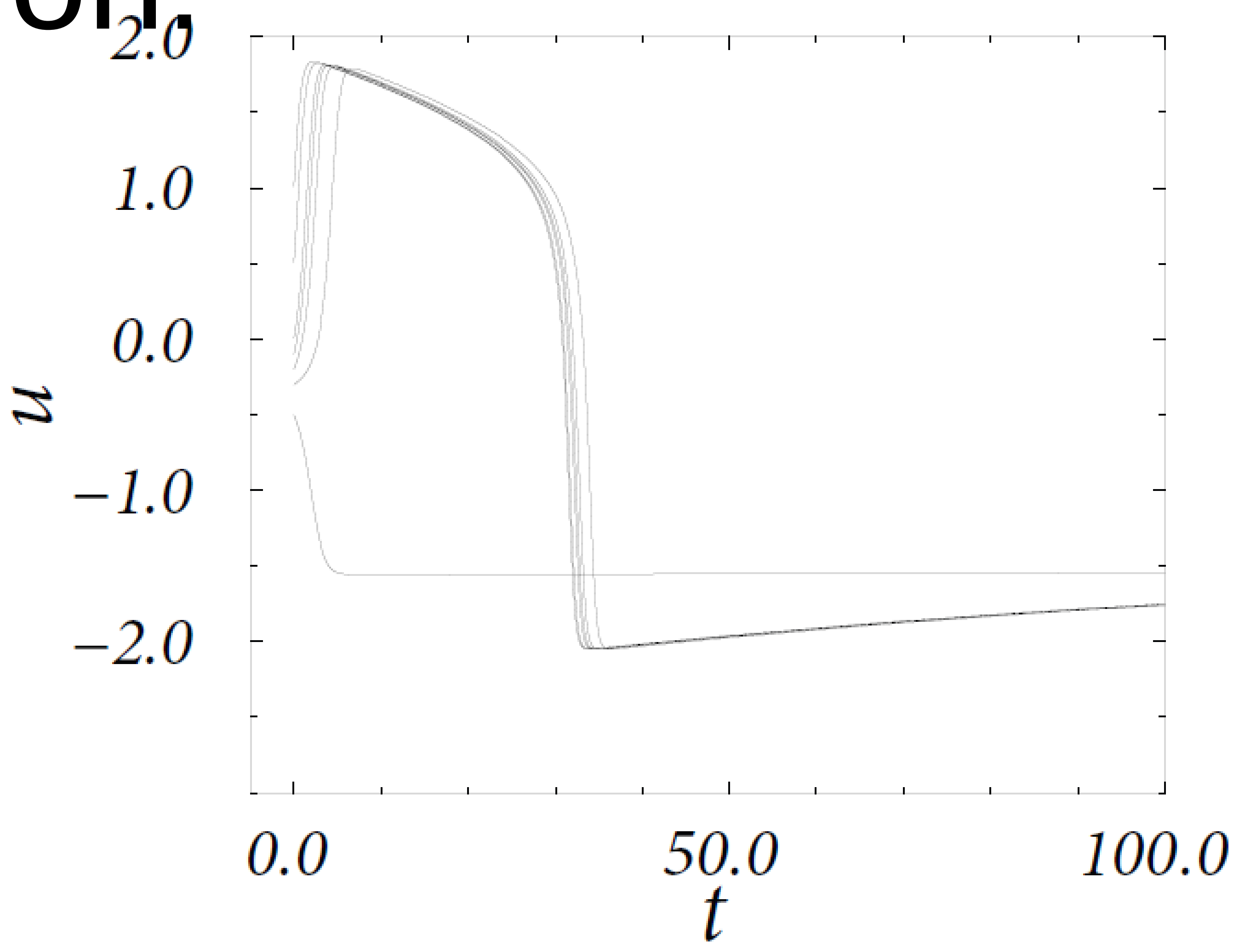


4.1 FitzHugh-Nagumo model: Threshold for Pulse input



Assumption:

$\tau_w >> \tau_u$



Middle branch of u-nullcline plays role of
‘Threshold’ (for pulse input)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

4.1 Detour: Separation fo time scales in 2dim models

stimulus

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

Assumption:

$$\tau_w >> \tau_u$$

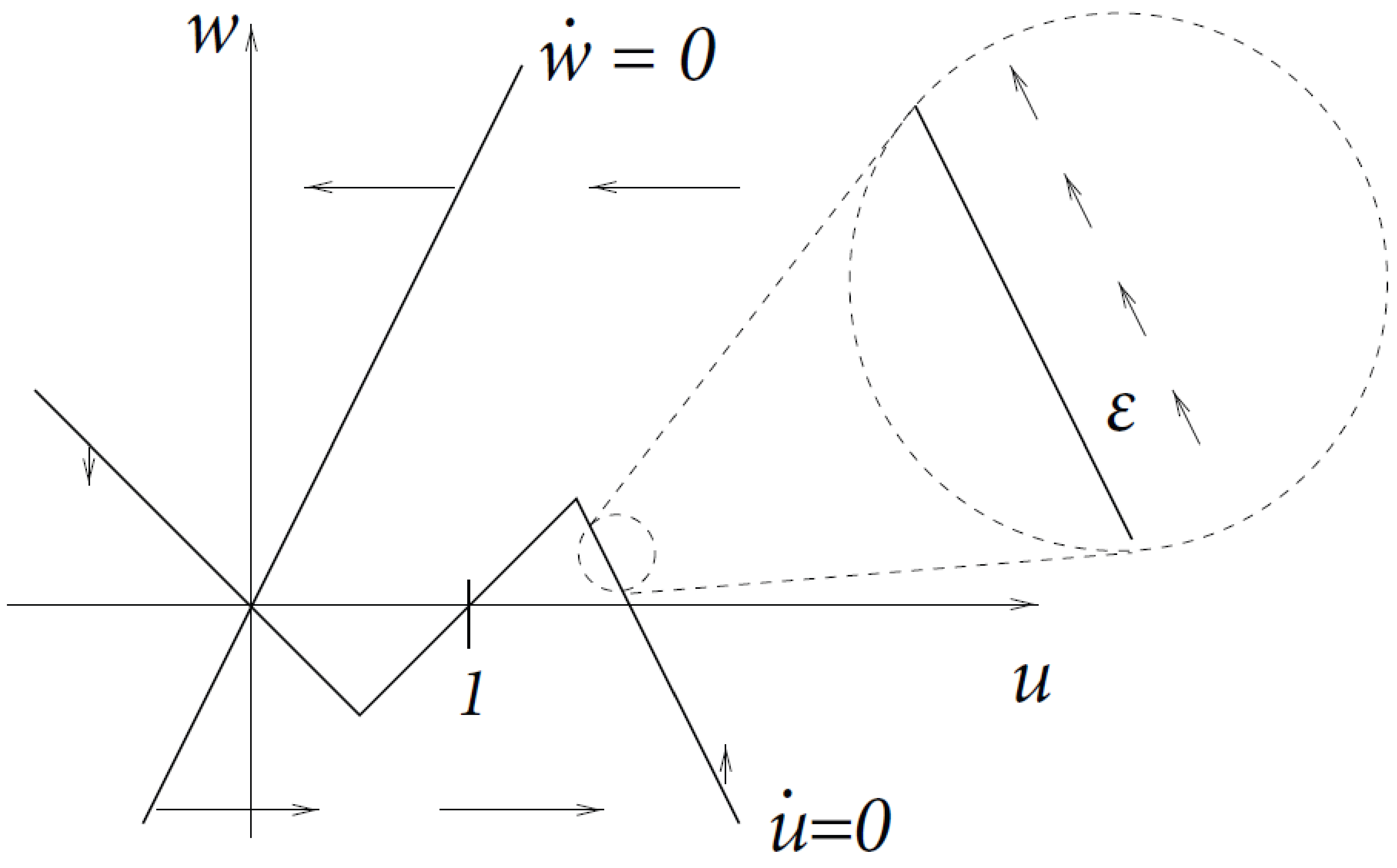
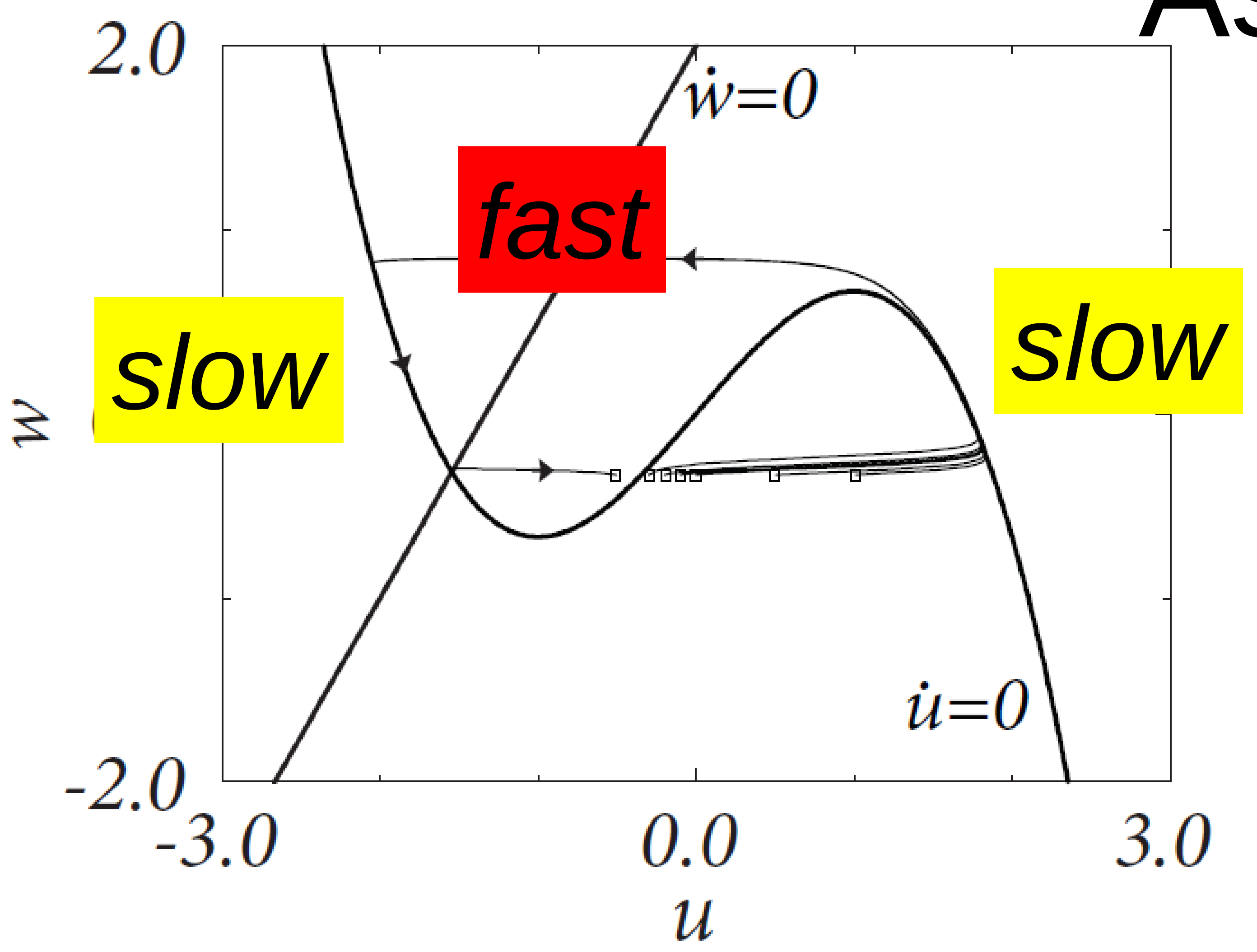


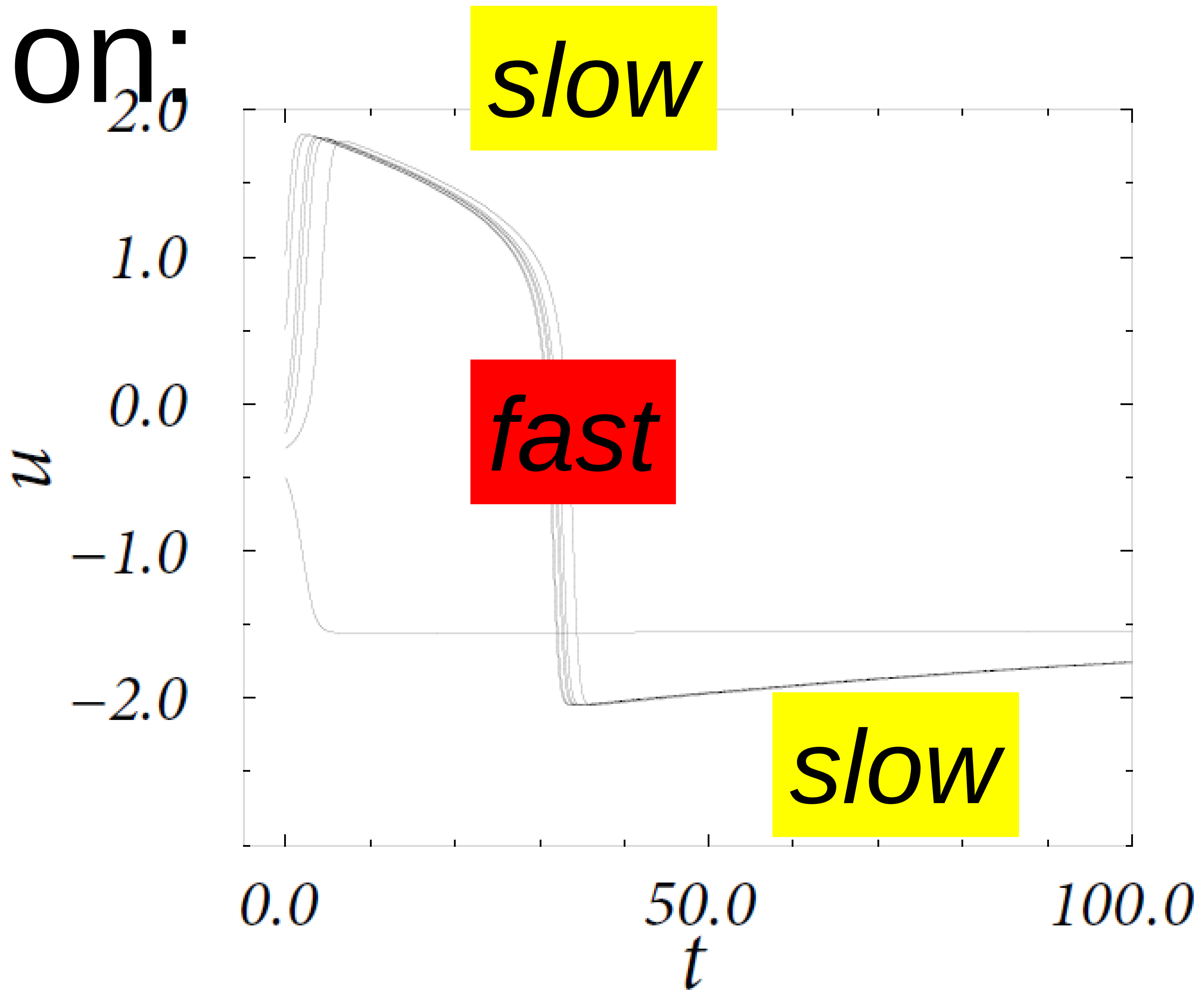
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

4.1 FitzHugh-Nagumo model: Threshold for Pulse input



Assumption:

$\tau_w >> \tau_u$



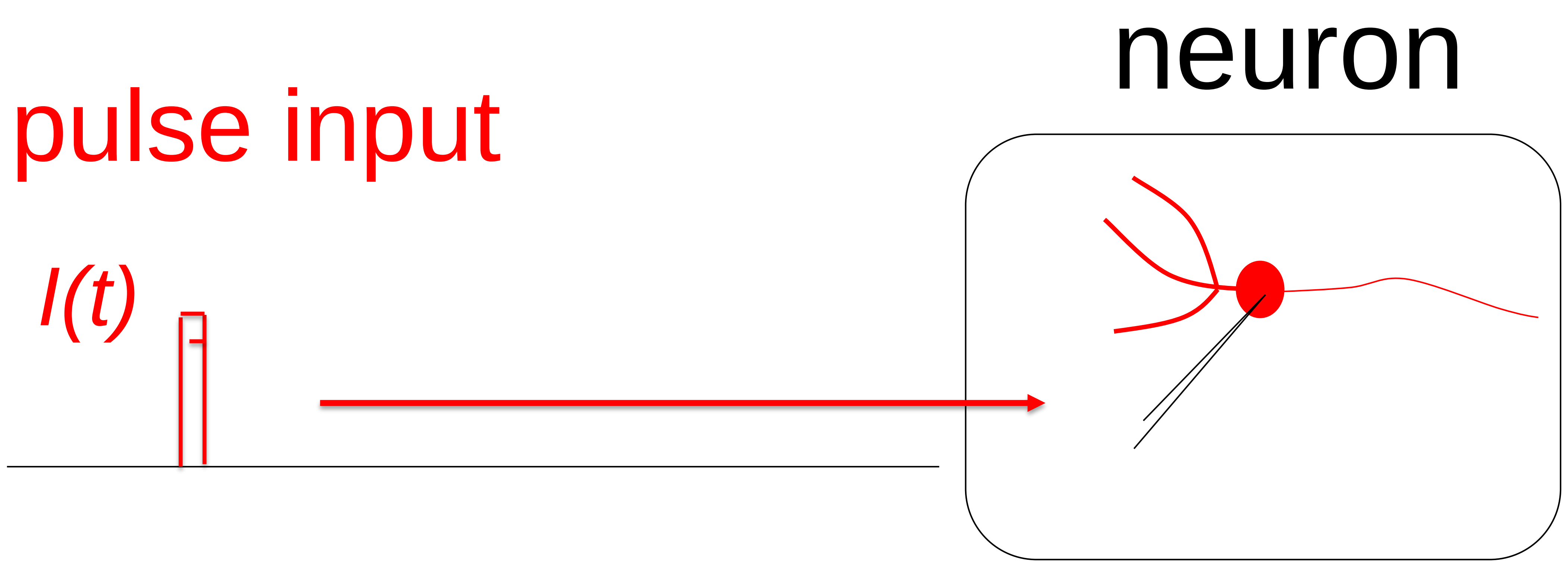
trajectory

- -follows u-nullcline: slow
- -jumps between branches: fast

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

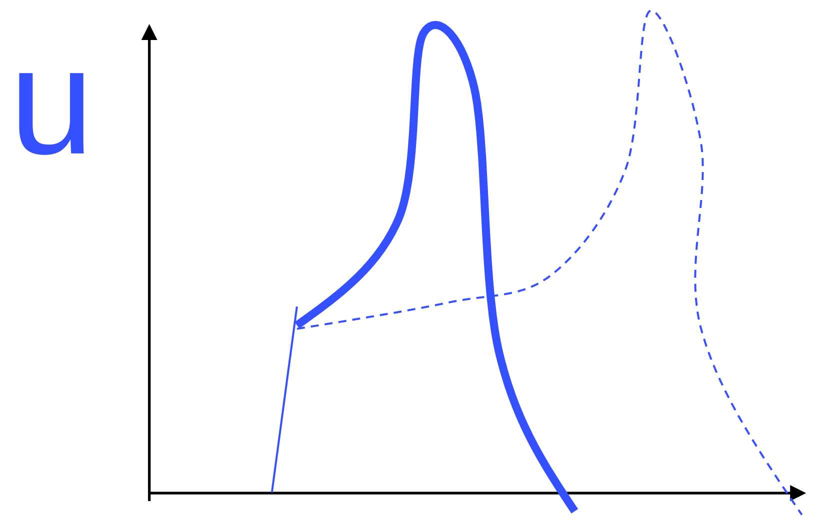
Biological input scenario

pulse input

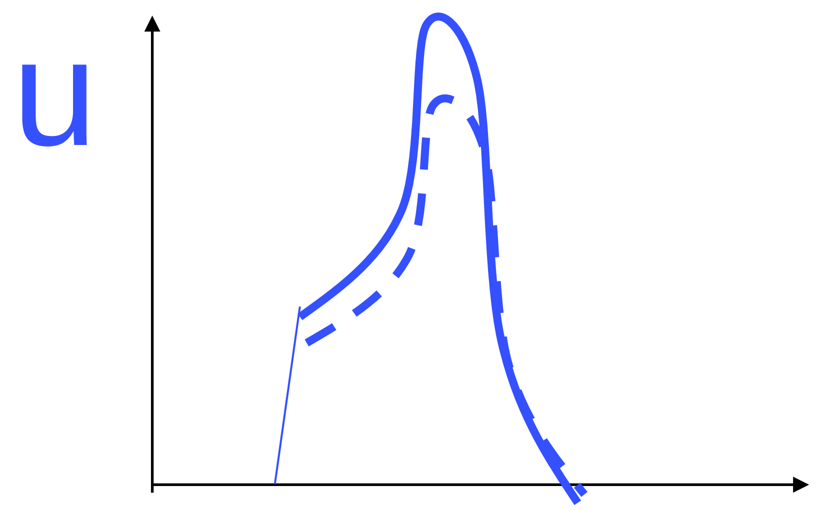


Mathematical explanation:
Graphical analysis in 2D

Delayed spike



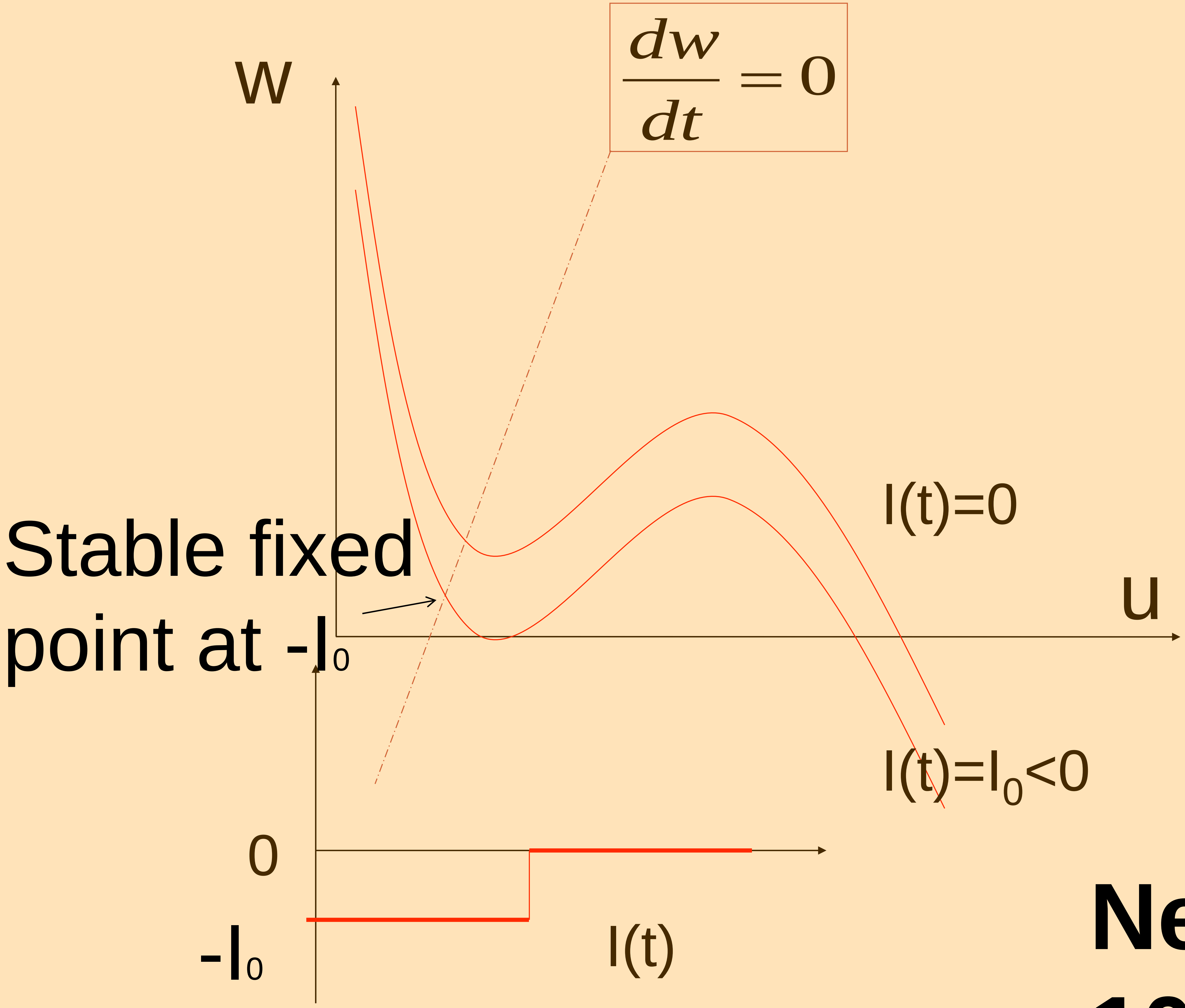
Reduced amplitude



Exercise 1: NOW!

inhibitory rebound

Assume separation of time scales



Next lecture: 10:55

Neuronal Dynamics – Literature for week 3 and 4.1

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski,
Neuronal Dynamics: from single neurons to networks and models of cognition. Chapter 4*: Introduction*. Cambridge Univ. Press, 2014
OR W. Gerstner and W.M. Kistler, *Spiking Neuron Models*, Ch.3. Cambridge 2002
OR J. Rinzel and G.B. Ermentrout, (1989). Analysis of neuronal excitability and oscillations. In Koch, C. Segev, I., editors, *Methods in neuronal modeling*. MIT Press, Cambridge, MA.

Selected references.

-Ermentrout, G. B. (1996). *Type I membranes, phase resetting curves, and synchrony*. Neural Computation, 8(5):979-1001.

-Fourcaud-Trocme, N., Hansel, D., van Vreeswijk, C., and Brunel, N. (2003). *How spike generation mechanisms determine the neuronal response to fluctuating input.* J. Neuroscience, 23:11628-11640.

-Badel, L., Lefort, S., Berger, T., Petersen, C., Gerstner, W., and Richardson, M. (2008). Biological Cybernetics, 99(4-5):361-370.

- E.M. Izhikevich, Dynamical Systems in Neuroscience, MIT Press (2007)

Neuronal Dynamics – Quiz 4.2.

A. Threshold in a 2-dimensional neuron model with saddle-node bifurcation

[ ] The voltage threshold for repetitive firing is always the same as the voltage threshold for pulse input.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for repetitive firing is given by the stable manifold of the saddle.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for repetitive firing is given by the middle branch of the u-nullcline.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for action potential firing in response to a short pulse input is given by the middle branch of the u-nullcline.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for action potential firing in response to a short pulse input is given by the stable manifold of the saddle point.

B. Threshold in a 2-dimensional neuron model with subcritical Hopf bifurcation

[ ]in the regime below the bifurcation, the voltage threshold for action potential firing in response to a short pulse input is given by the stable manifold of the saddle point.

[ ] in the regime below the bifurcation, a voltage threshold for action potential firing in response to a short pulse input exists only if $\tau_w >> \tau_u$