9.1 Firing patterns and adaptation

9.2 AdEx model
   - Firing patterns and adaptation

9.3 Spike Response Model (SRM)
   - Integral formulation
Step current input – neurons show adaptation

Data: Markram et al. (2004)

1-dimensional (nonlinear) integrate-and-fire model cannot do this!
Firing patterns:

$I(t)$
Biological Modeling of Neural Networks:

Week 9 – Adaptation and firing patterns

Wulfram Gerstner
EPFL, Lausanne, Switzerland

9.1 Firing patterns and adaptation
9.2 AdEx model
- Firing patterns and adaptation
9.3 Spike Response Model (SRM)
- Integral formulation
Add adaptation variables:

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \theta}{\Delta}\right) - R \sum_k w_k \]

\[ \tau_k \frac{dw_k}{dt} = a_k (u - u_{\text{rest}}) - w_k + b_k \tau_k \sum_f \delta(t - t^f) \]

SPIKE AND RESET

- after each spike \( w_k \) jumps by an amount \( b_k \)
- If \( u = \theta_{\text{reset}} \) then reset to \( u = u_r \)

AdEx model, Brette&Gerstner (2005):

Exponential I&F
+ 1 adaptation var.
= AdEx

Blackboard!
Firing patterns:
Response to Step currents,
Exper. Data, Markram et al. (2004)
$I(t)$
Firing patterns:
Response to Step currents, AdEx Model, Naud & Gerstner

Neuronal Dynamics – 9.2 Adaptive Exponential I&F

\[
\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = a \ (u - u_{rest}) - w + b \ \tau_w \sum_{f} \delta(t - t^f)
\]

AdEx model

Phase plane analysis!

Can we understand the different firing patterns?
Neuronal Dynamics – Quiz 9.1. Nullclines of AdEx

\[ \tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \theta}{\Delta}\right) - Rw + RI(t) \]
\[ \tau_w \frac{dw}{dt} = a \ (u - u_{rest}) - w \]

A - What is the qualitative shape of the w-nullcline?

[ ] constant
[ ] linear, slope a
[ ] linear, slope 1
[ ] linear + quadratic
[ ] linear + exponential

B - What is the qualitative shape of the u-nullcline?

[ ] linear, slope 1
[ ] linear, slope 1/R
[ ] linear + quadratic
[ ] linear w. slope 1/R + exponential

1 minute
Restart at 9:38
9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM

9.5 Parameter Estimation
- Quadratic and convex optimization

9.6 Modeling in vitro data
- How long lasts the effect of a spike?
AdEx model

after each spike
u is reset to $u_r$

\[
\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t)
\]

after each spike
w jumps by an amount $b$

\[
\tau_w \frac{dw}{dt} = a \ (u - u_{rest}) - w + b \ \tau_w \sum_f \delta(t - t^f)
\]

parameter $a$ – slope of $w$-nullcline

Can we understand the different firing patterns?
AdEx model – phase plane analysis: large $b$

\[
\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \theta}{\Delta}\right) + w + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = a (u - u_{rest}) - w + b \tau_w \sum_f \delta(t - t^f)
\]

$a=0$

A

B

$u$ is reset to $u_r$
AdEx model – phase plane analysis: small $b$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \varrho}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a (u - u_{rest}) - w + b \tau_w \sum_{f} \delta(t - t^f)$$

adaptation

$u$ is reset to $u_r$
Quiz 9.2: AdEx model – phase plane analysis

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \theta}{\Delta}\right) + w + RI(t) \]

\[ \tau_w \frac{dw}{dt} = a (u - u_{\text{rest}}) + b \tau_w \sum_f \delta(t - t_f) \]

What firing pattern do you expect?

(i) Adapting
(ii) Bursting
(iii) Initial burst
(iv) Non-adapting

\[ u \text{ is reset to } u_r \]
AdEx model – phase plane analysis: $a > 0$

\[
\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp \left( \frac{u - \vartheta}{\Delta} \right) + w + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = a \left( u - u_{\text{rest}} \right) - w + b \tau_w \sum_{f} \delta(t - t^f)
\]
Neuronal Dynamics – 9.2 AdEx model and firing patterns

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t) \]

\[ \tau_\omega \frac{dw}{dt} = a(u - u_{\text{rest}}) - w + b \tau_\omega \sum_f \delta(t - t^f) \]

- after each spike \( u \) is reset to \( u_r \)
- after each spike \( w \) jumps by an amount \( b \)
- parameter \( a \) – slope of \( w \) nullcline

Firing patterns arise from different parameters!

See Naud et al. (2008), see also Izikhevich (2003)
Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) \[ \tau \frac{du}{dt} = f(u) + R I(t) \]

(2) If \( u = \theta_{\text{reset}} \) then reset to \( u = u_r \)

Best choice of \( f \): linear + exponential

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \mathcal{J}}{\Delta}\right) \]

BUT: Limitations – need to add

✓ -Adaptation on slower time scales
✓ -Possibility for a diversity of firing patterns
✓ -Increased threshold \( \mathcal{J} \) after each spike
✓ -Noise
Neuronal Dynamics – 9.2 AdEx with dynamic threshold

Add dynamic threshold:

$$\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - R \sum_k w_k + RI(t)$$

Threshold increases after each spike

$$\vartheta = \theta_0 + \sum_f \theta_1(t - t^f)$$
Neuronal Dynamics – 9.2 Generalized Integrate-and-fire

\[ \tau \frac{du}{dt} = f(u) + R I(t) \]

If \( u = \theta_{\text{reset}} \) then reset to \( u = u_r \)

**add**

- Adaptation variables
- Possibility for firing patterns
- Dynamic threshold \( \mathcal{D} \)
- Noise
Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Week 9 – part 3: Spike Response Model (SRM)

- 9.1 What is a good neuron model?
  - Models and data
- 9.2 AdEx model
  - Firing patterns and adaptation
- 9.3 Spike Response Model (SRM)
  - Integral formulation
- 9.4 Generalized Linear Model
  - Adding noise to the SRM
- 9.5 Parameter Estimation
  - Quadratic and convex optimization
- 9.6 Modelin in vitro data
  - how long lasts the effect of a spike?
Exponential versus Leaky Integrate-and-Fire

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \mathcal{G}}{\Delta}\right) + RI(t) \]

Reset if \( u = \mathcal{G} \)

Leaky Integrate-and-Fire:
Replace nonlinear kink by threshold

Neuronal Dynamics – 9.3 Adaptive leaky integrate-and-fire

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) - R \sum_k w_k + RI(t) \]

\[ \tau_k \frac{dw_k}{dt} = a_k (u - u_{\text{rest}}) - w_k + b_k \tau_k \sum_f \delta(t - t^f) \]

SPIKE AND RESET

- after each spike \( w_k \) jumps by an amount \( b_k \)
- If \( u = \mathcal{I}(t) \) then reset to \( u = u_r \)

Dynamic threshold
Neuronal Dynamics – 9.3 Adaptive leaky I&F and SRM

\[
\tau \frac{du}{dt} = -(u - u_{rest}) - R \sum_k w_k + RI(t)
\]
\[
\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)
\]

Linear equation \(\rightarrow\) can be integrated!

\[
u(t) = \sum_f \eta(t - t^f) + \int_0^\infty ds \kappa(s) I(t - s)
\]
\[
\theta(t) = \theta_0 + \sum_f \theta_1(t - t^f)
\]

Adaptive leaky I&F

Spike Response Model (SRM)

Gerstner et al. (1996)
Neuronal Dynamics – 9.3 Spike Response Model (SRM)

Gerstner et al., 1993, 1996

Input

$\varphi(t)$

$u(t)$

Spike emission

Arbitrary Linear filters

Potential

$u(t) = \sum_{t'} \eta(t-t') + \int_0^\infty \kappa(s) I(t-s) ds + u_{\text{rest}}$

Threshold

$\vartheta(t) = \theta_0 + \sum_{t'} \theta_1(t-t')$
SRM with appropriate $\eta$ leads to bursting

\[
\begin{align*}
  u(t) &= \sum_{f} \eta (t - t_f^f) + \int_{0}^{\infty} ds \, \kappa(s) I(t - s) + u_{\text{rest}} \\
  u(t) &= \int_{0}^{\infty} ds \, \eta(s) S(t - s) + \int_{0}^{\infty} ds \, \kappa(s) I(t - s) + u_{\text{rest}}
\end{align*}
\]
Exercise 1: from adaptive IF to SRM

\[
\tau \frac{du}{dt} = -(u - u_{\text{rest}}) - w + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = -w + b \tau_w \sum_f \delta(t - t^f)
\]

If \( u = 0 \) then reset to \( u = u_r \)

Integrate the above system of two differential equations so as to rewrite the equations as

potential

\[
u(t) = \int_0^\infty \eta(s) S(t - s) \, ds + \int_0^\infty \varepsilon(s) I(t - s) \, ds + u_{\text{rest}}
\]

A – what is \( \eta(s) \)?

(i) \( x(s) = \frac{R}{\tau} \exp\left(-\frac{s}{\tau}\right) \)

(ii) \( x(s) = \frac{R}{\tau_w} \exp\left(-\frac{s}{\tau_w}\right) \)

B – what is \( \varepsilon(s) \)?

(iii) \( x(s) = C[\exp\left(-\frac{s}{\tau}\right) - \exp\left(-\frac{s}{\tau_w}\right)] \)

(iv) Combi of (i) + (iii)

Next lecture at 9:57/10:15
Neuronal Dynamics – 9.3 Spike Response Model (SRM)

Potential
\[ u(t) = \sum_{t'} \eta(t - t') + \int_0^\infty \kappa(s) I(t - s) ds + u_{rest} \]

Threshold
\[ \vartheta(t) = \theta_0 + \sum_{t'} \theta_1(t - t') \]

Firing if
\[ u(t) = \vartheta(t) \]

Gerstner et al., 1993, 1996
Neuronal Dynamics – 9.3 Spike Response Model (SRM)

potential

$$u(t) = \sum_{t'} \eta(t-t') + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

threshold

$$\theta(t) = \theta_0 + \sum_{t'} \theta_1(t-t')$$

Linear filters for
- input
- threshold
- refractoriness