

# Week 6: Hebbian Learning



## Biological Modeling of Neural Networks

### Week 6

### Hebbian LEARNING and ASSOCIATIVE MEMORY

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#### 6.1 Synaptic Plasticity

- Hebbian Learning
- Short-term Plasticity
- Long-term Plasticity
- Reinforcement Learning

#### 6.2 Models of synaptic plasticity

- Hebbian learning rules

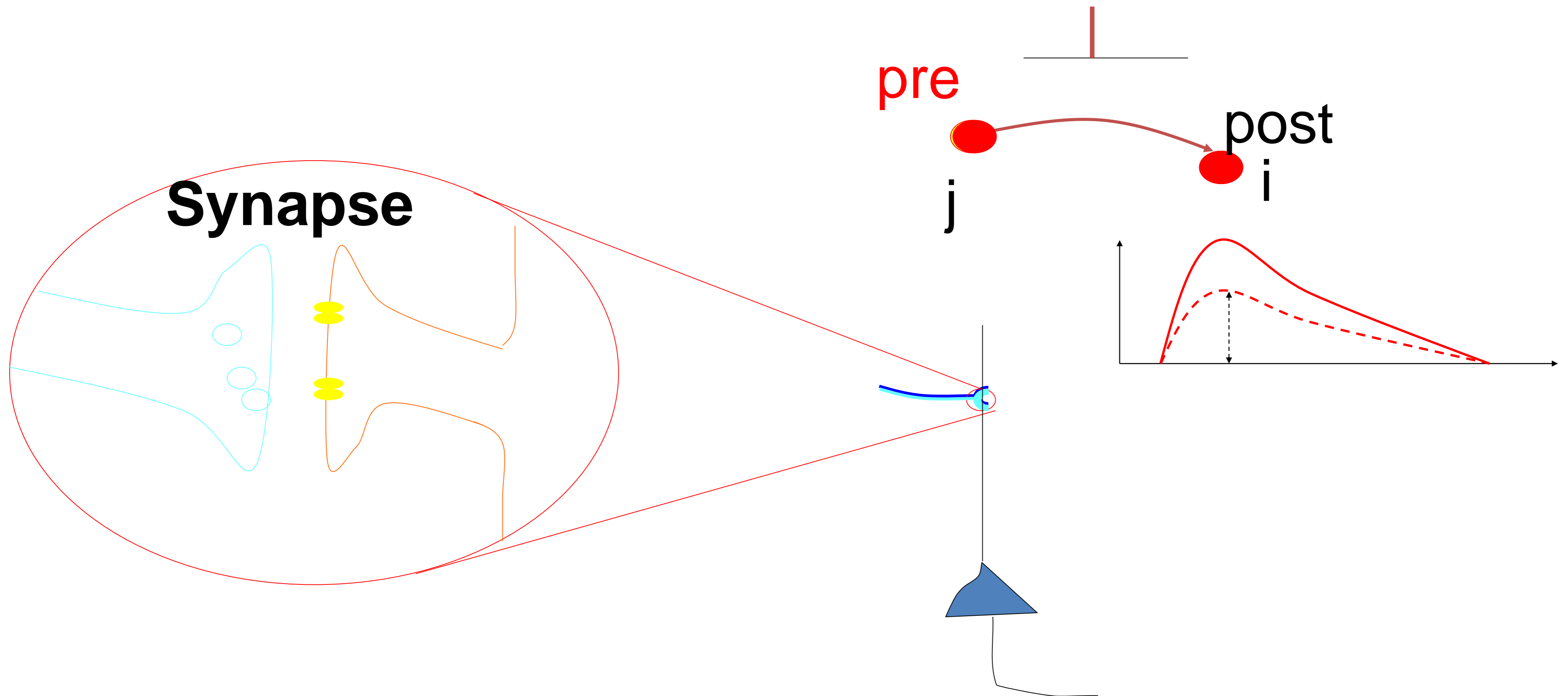
#### 6.3

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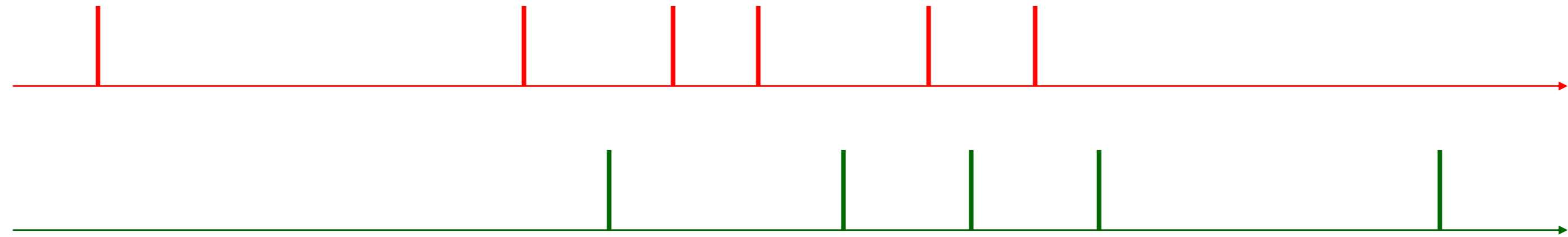
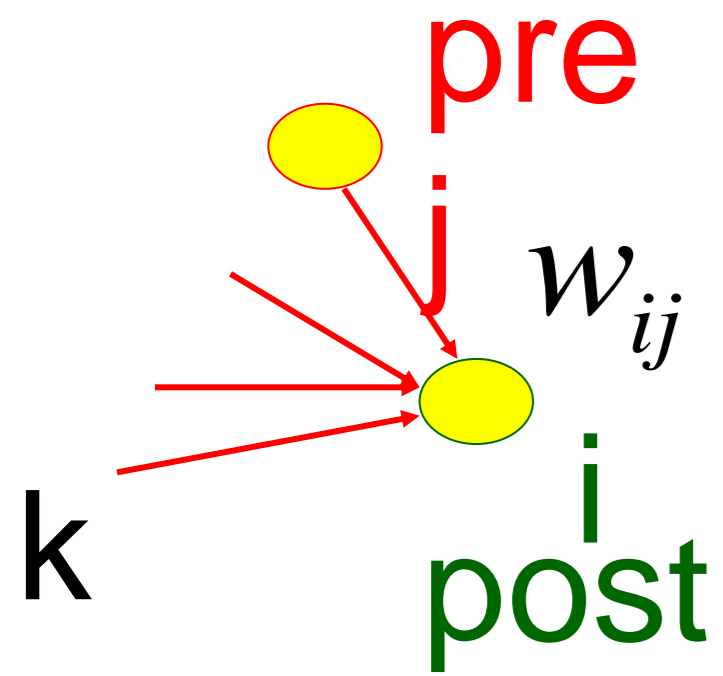
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#### 6.4

# 6.1 Synaptic plasticity



# 6.1 Synaptic plasticity: Hebbian Learning



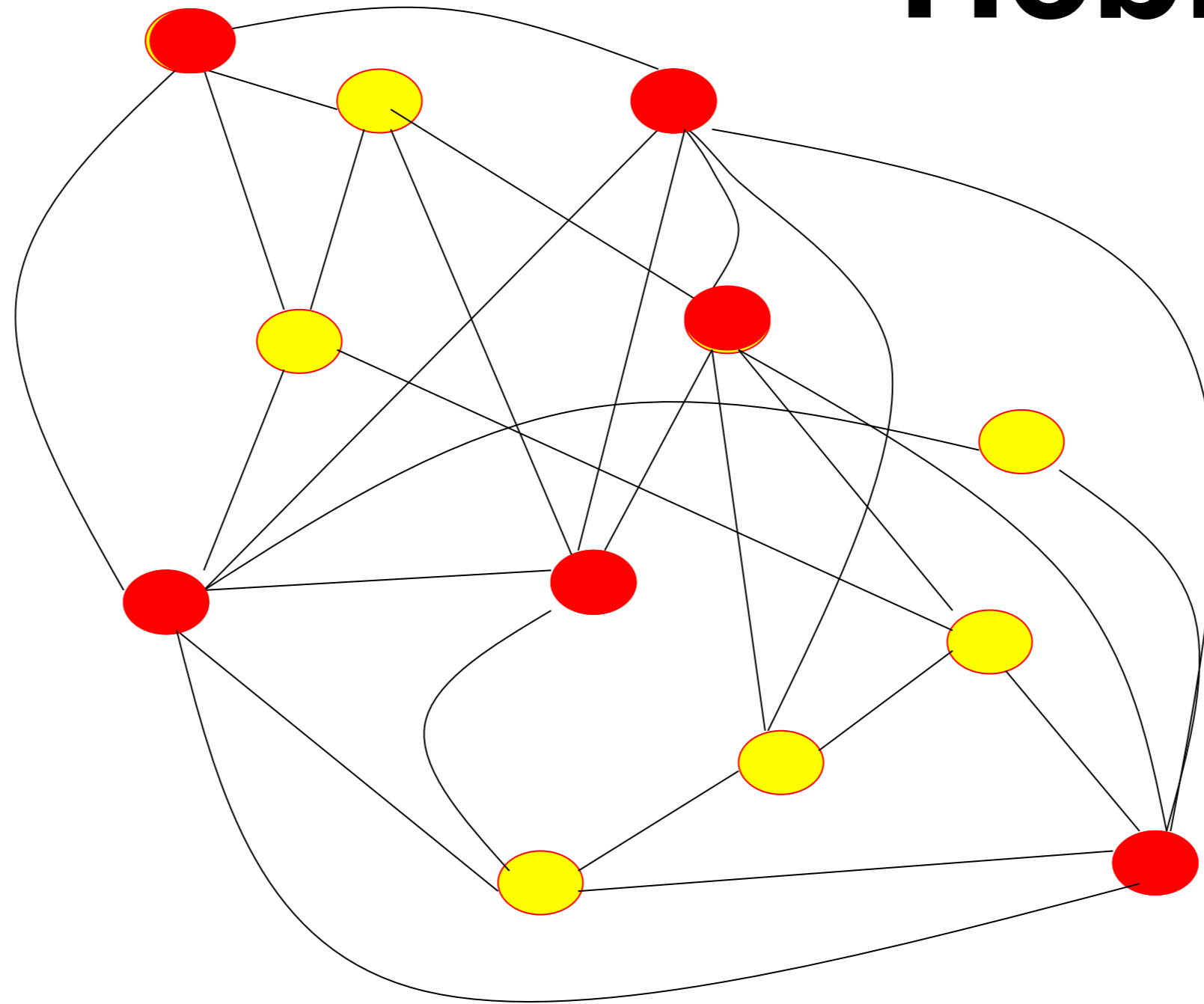
When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then **j**'s efficiency as one of the cells firing **i** is increased

Hebb, 1949

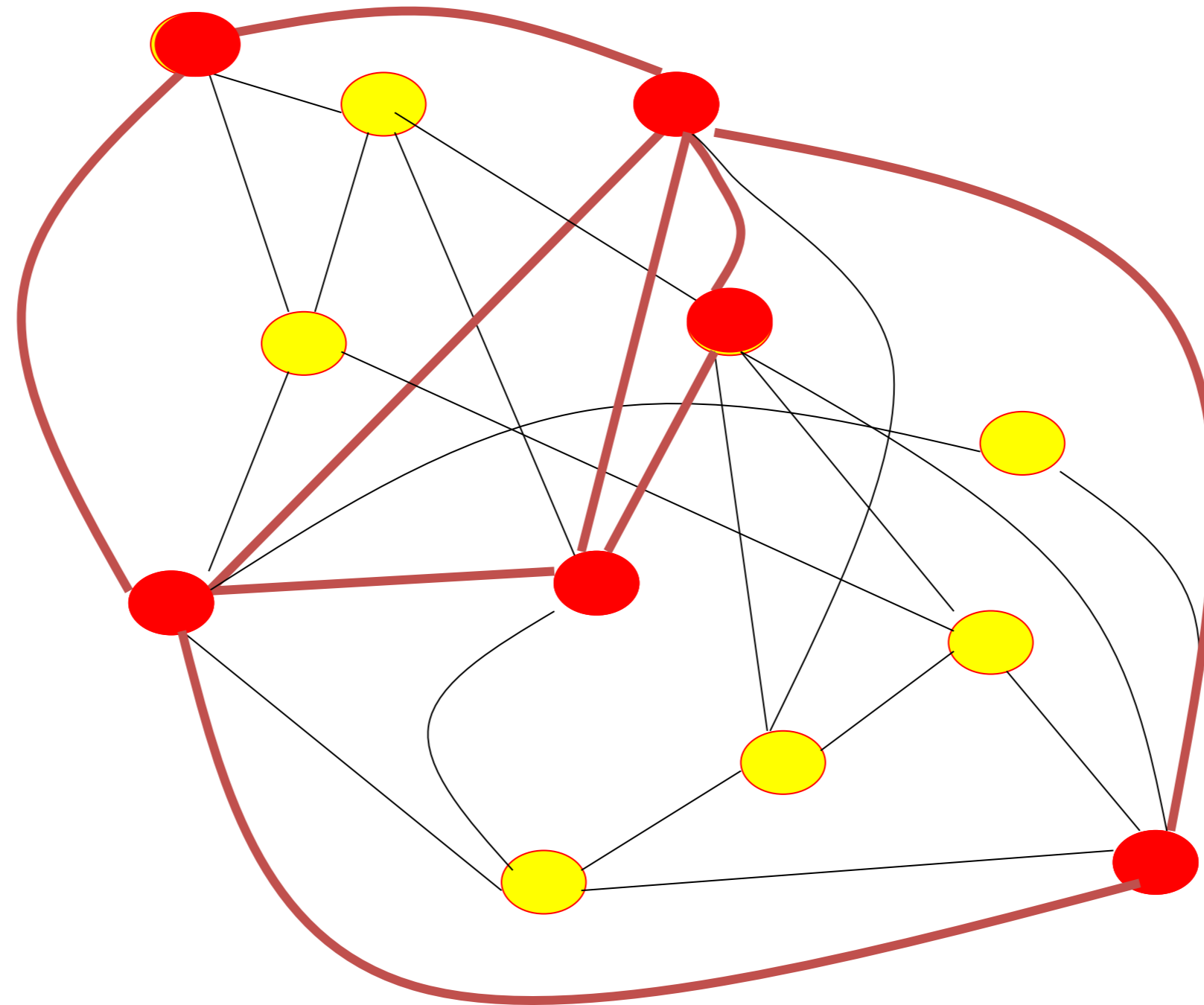
- local rule
- simultaneously active (correlations)

# 6.1 Synaptic plasticity: Hebbian Learning

## Hebbian Learning



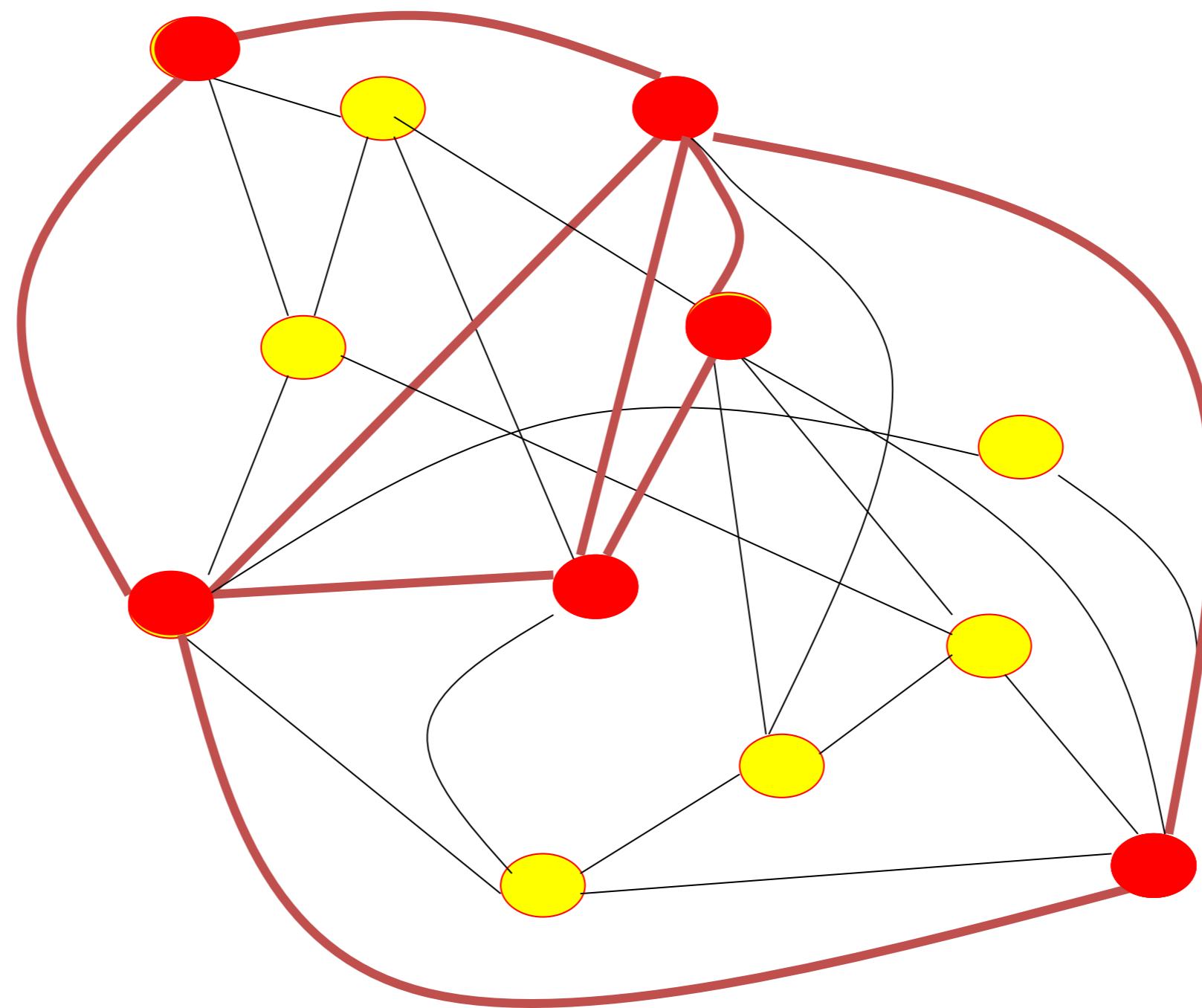
# 6.1 Synaptic plasticity: Hebbian Learning



item memorized

# 6.1 Synaptic plasticity: Hebbian Learning

Recall:  
Partial info



item recalled

# 6.1 Synaptic plasticity

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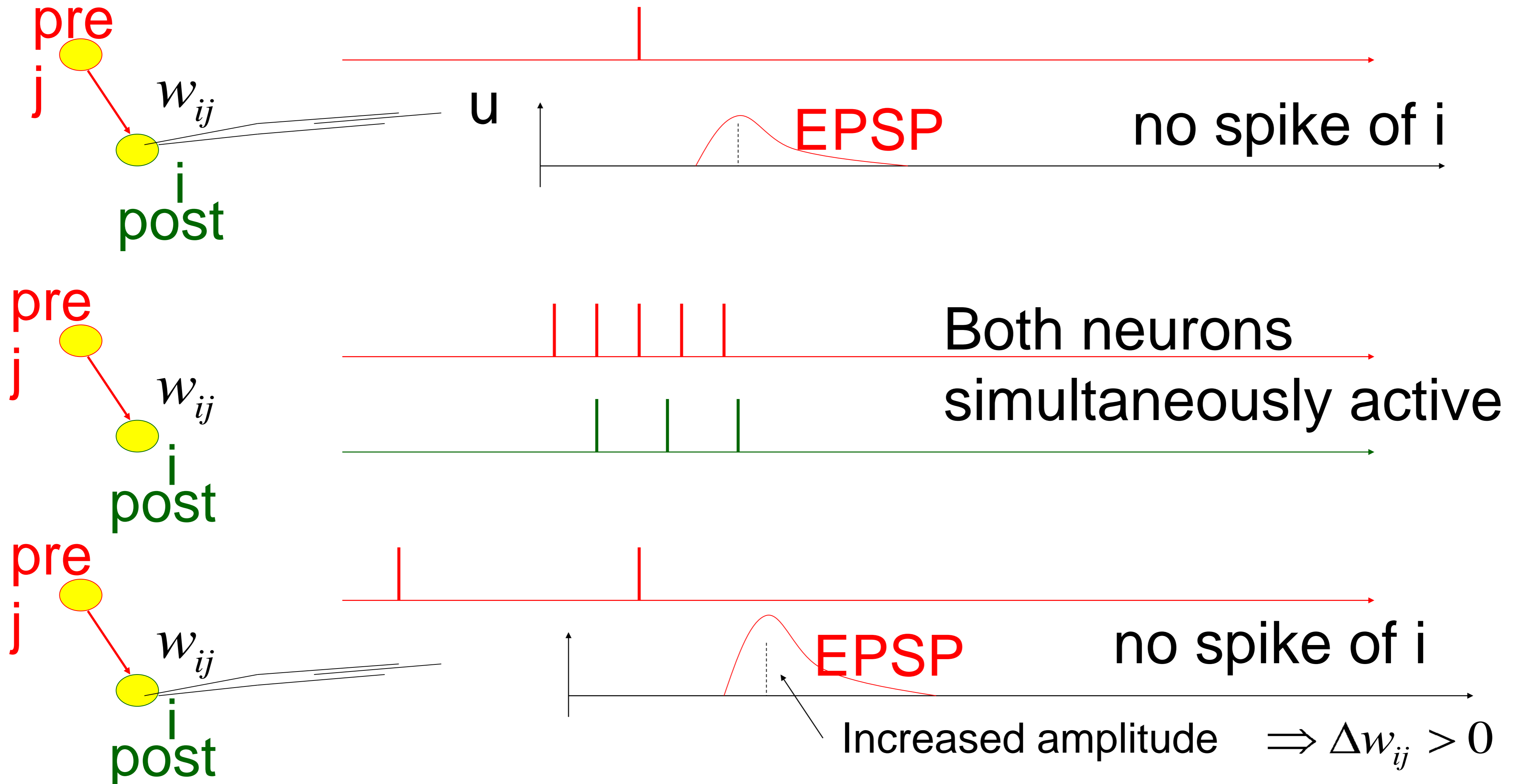
- Hebbian Learning

-  - Experiments on synaptic plasticity

- Formulations of Hebbian Learning

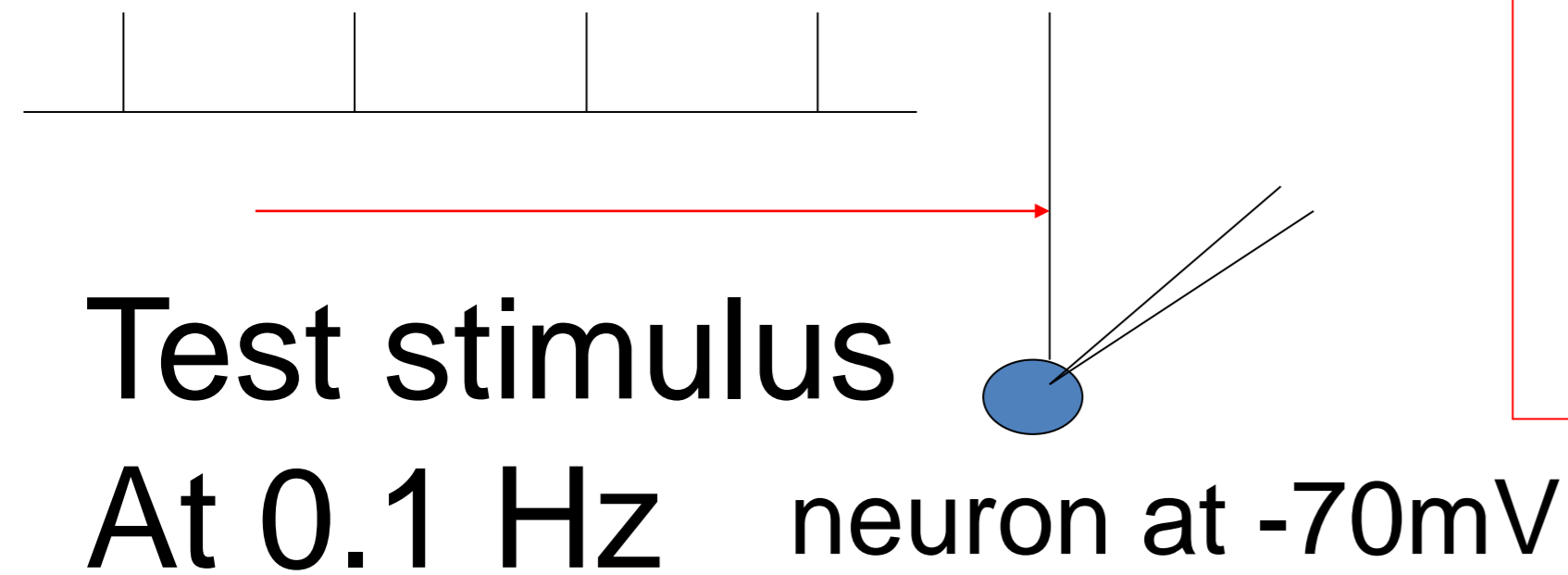
# 6.1 Synaptic plasticity

## Hebbian Learning in experiments (schematic)





# Classical paradigm of LTP induction – pairing



LTP induction:  
tetanus at 100Hz

neuron depolarized  
to -40mV

Standard LTP  
PAIRING experiment

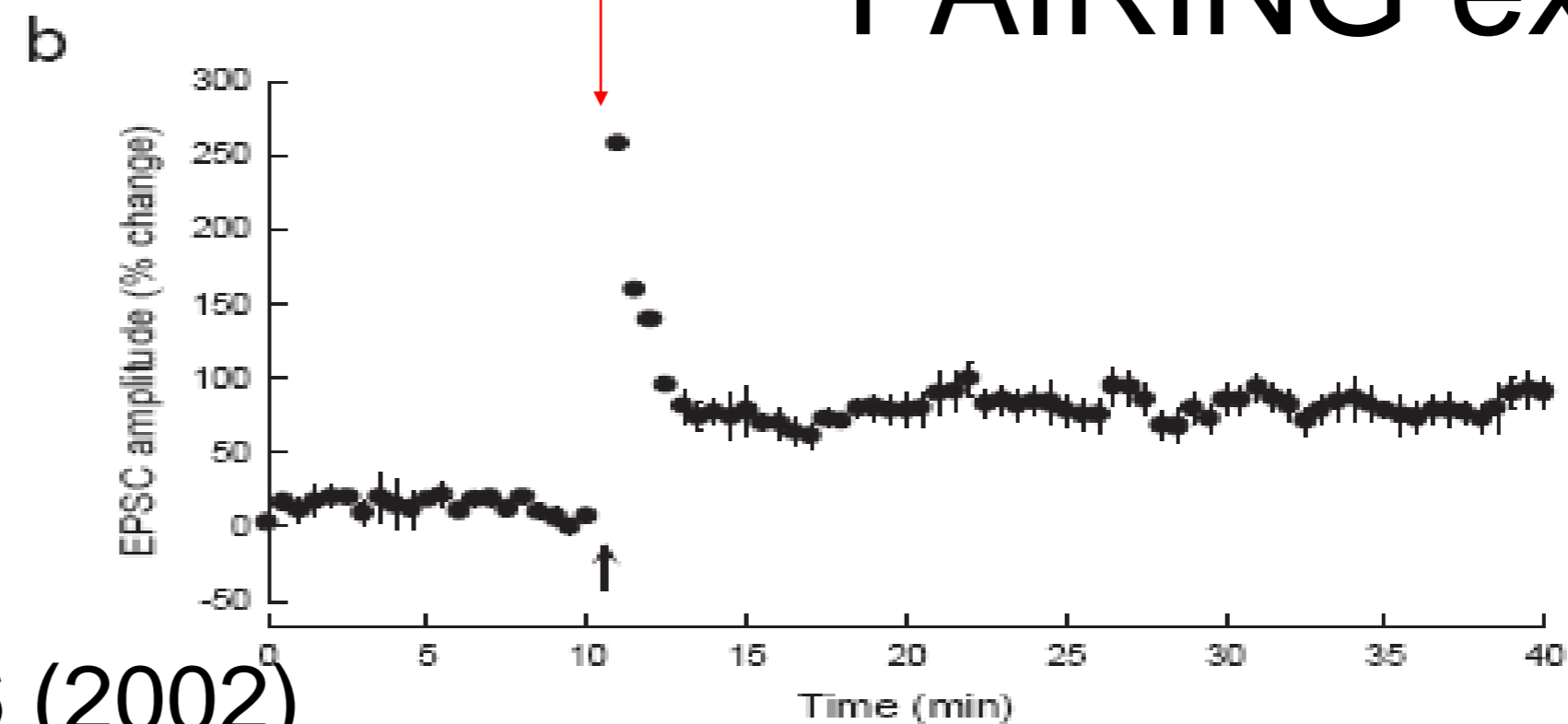
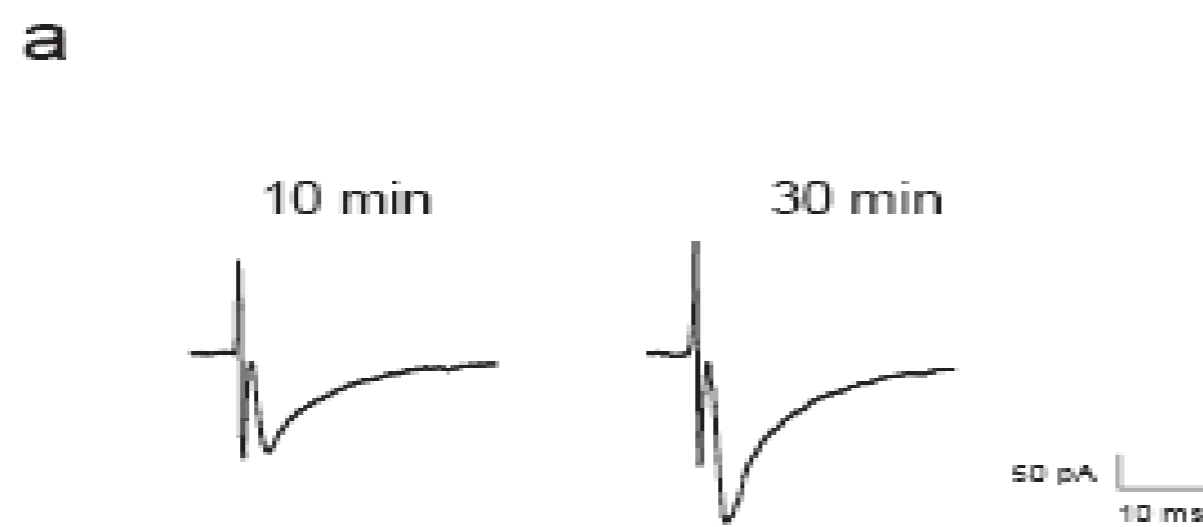
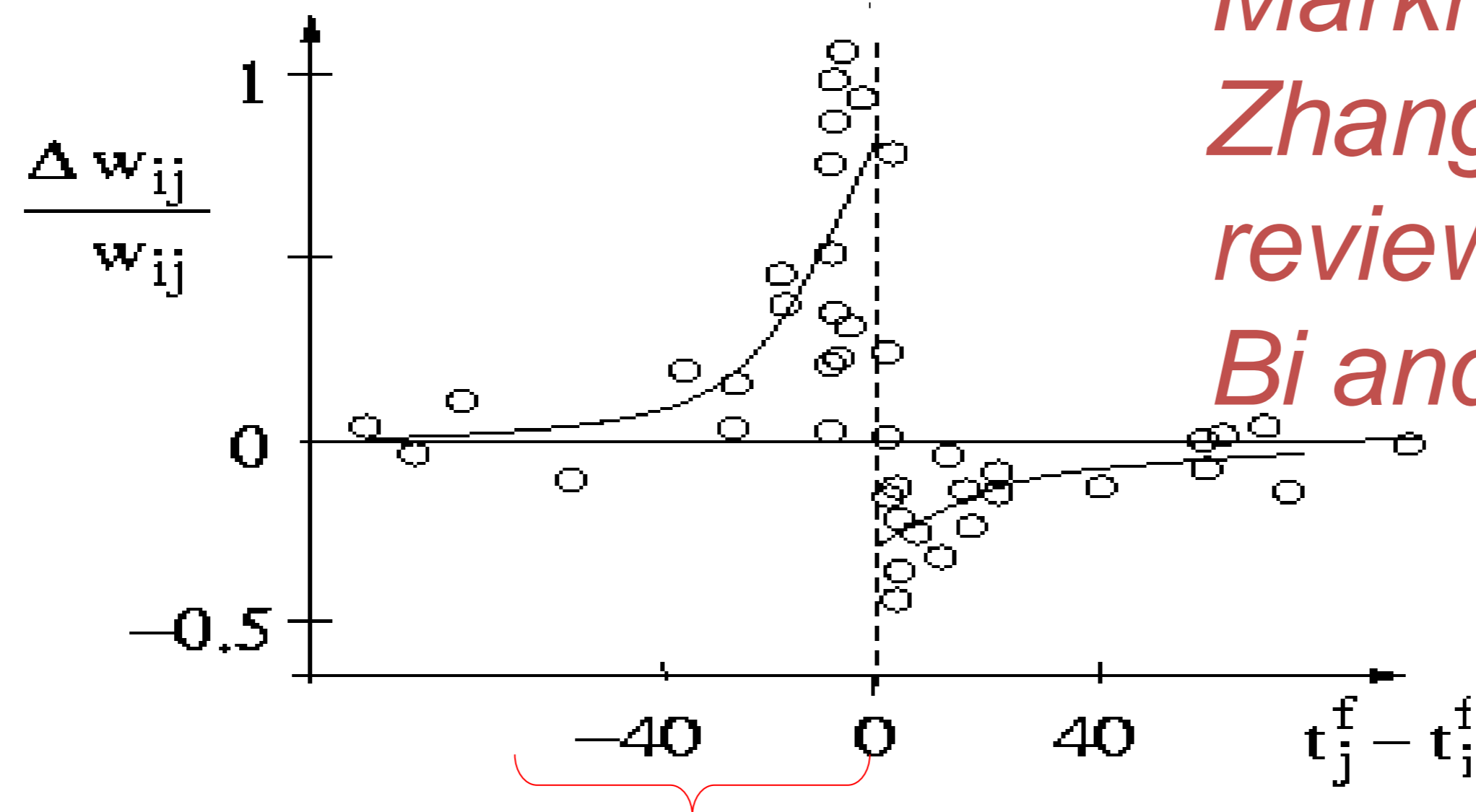
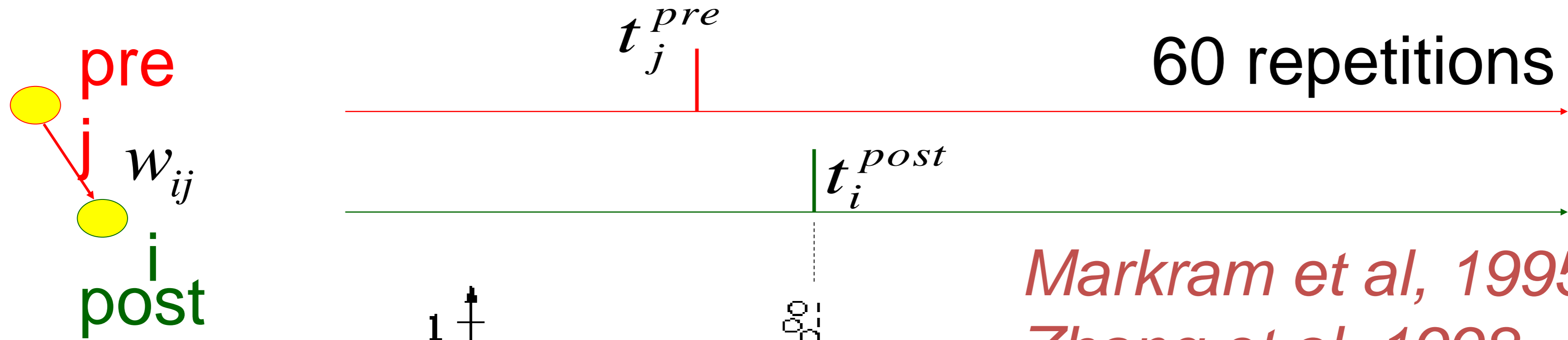


Fig. from Nature Neuroscience **5**, 295 - 296 (2002)

D. S.F. Ling, ... & Todd C. Sacktor

See also: Bliss and Lomo (1973), Artola, Brocher, Singer (1990), Bliss and Collingridge (1993)

# Spike-timing dependent plasticity (STDP)



*Markram et al, 1995, 1997*  
*Zhang et al, 1998*  
*review:*  
*Bi and Poo, 2001*

Pre  
before post

# 6.1 Classification of synaptic changes

## Induction of changes

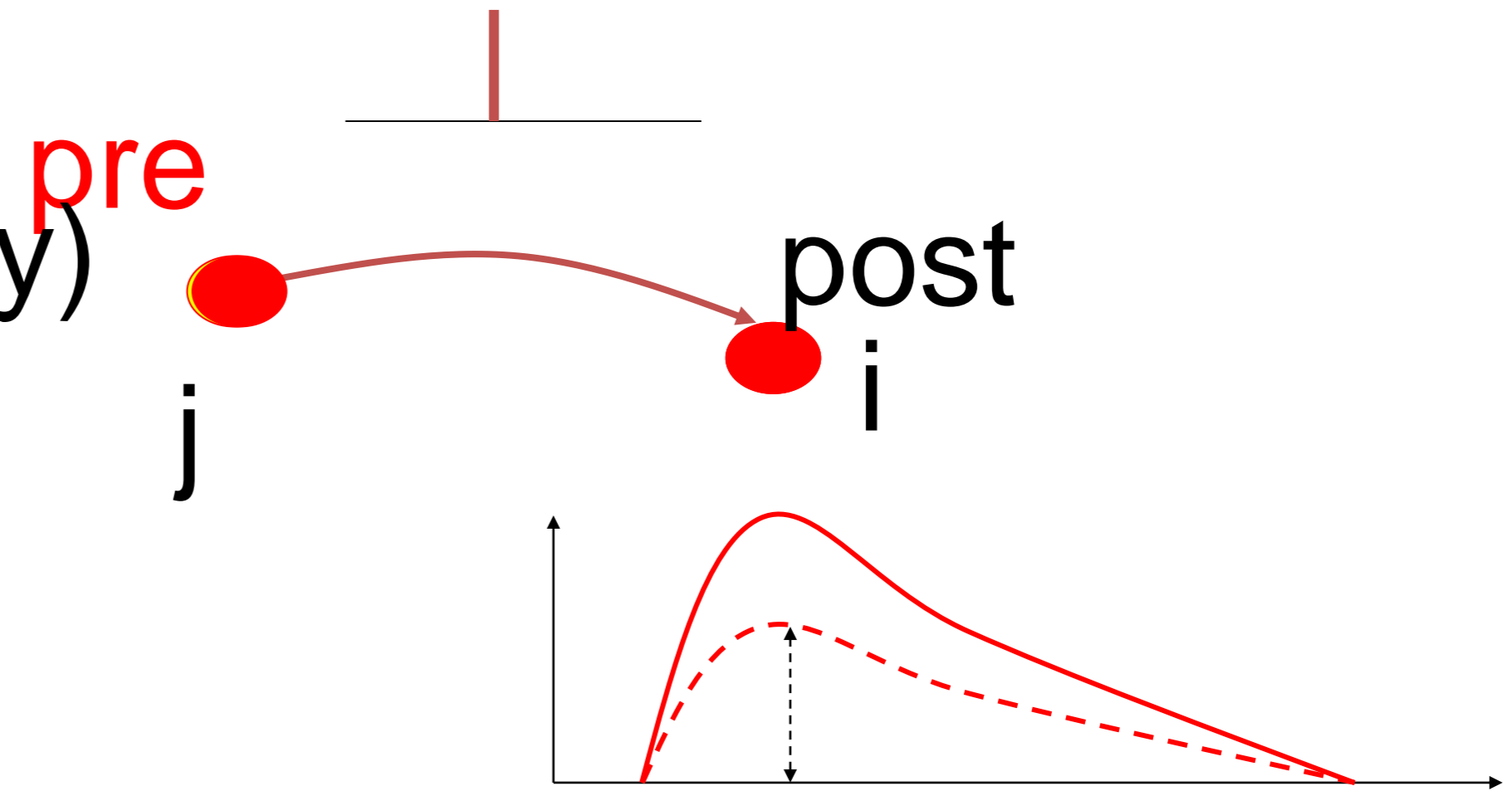
- fast (if stimulated appropriately)
- slow (homeostasis)

## Persistence of changes

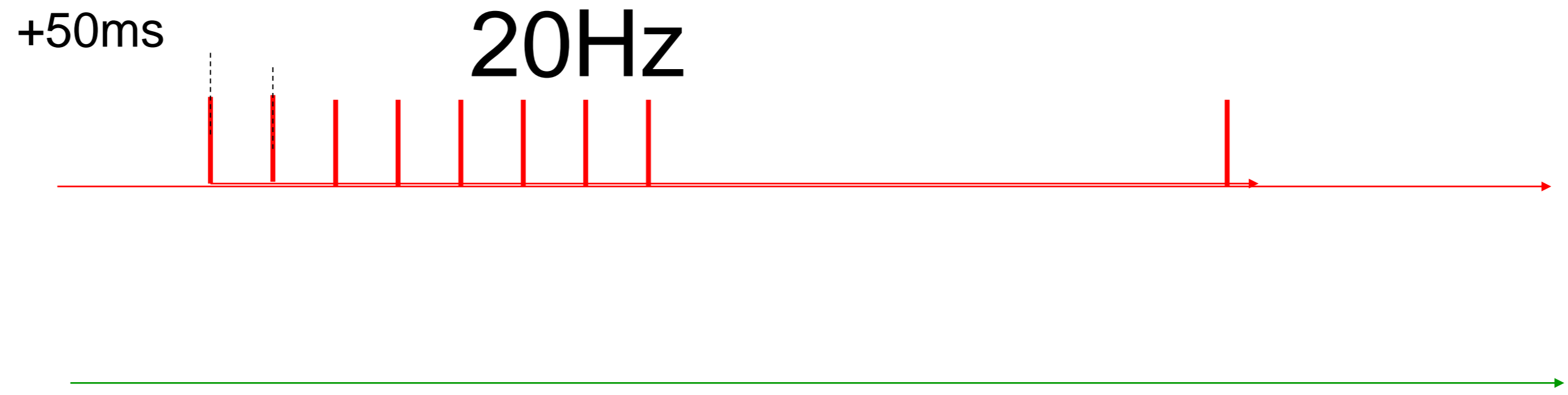
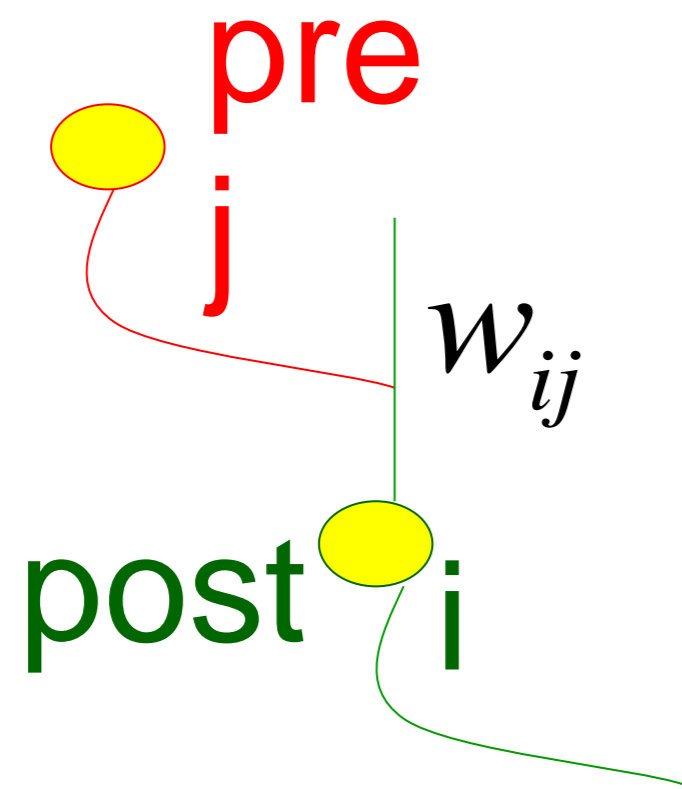
- long (LTP/LTD)
- short (short-term plasticity)

## Functionality

- useful for learning a new behavior
- useful for development (wiring for receptive field development)
- useful for activity control in network
- useful for coding



# 6.1 Classification of synaptic changes: Short-term plasticity

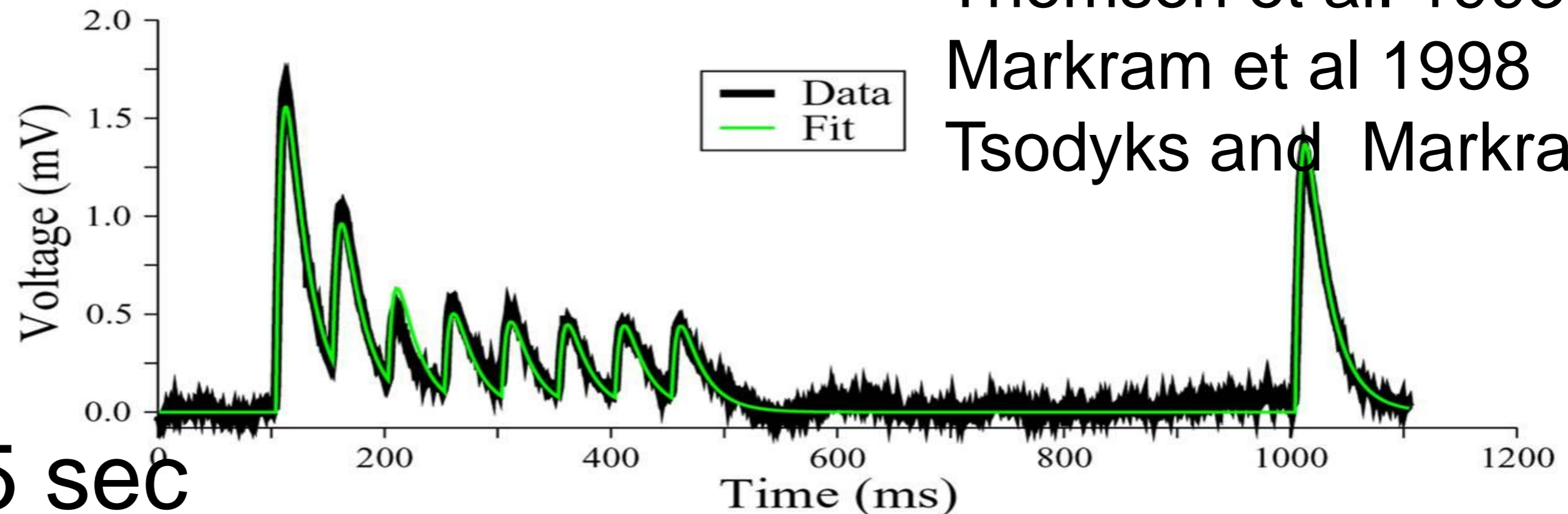


## Short-term plasticity/fast synaptic dynamics

Thomson et al. 1993

Markram et al 1998

Tsodyks and Markram 1997



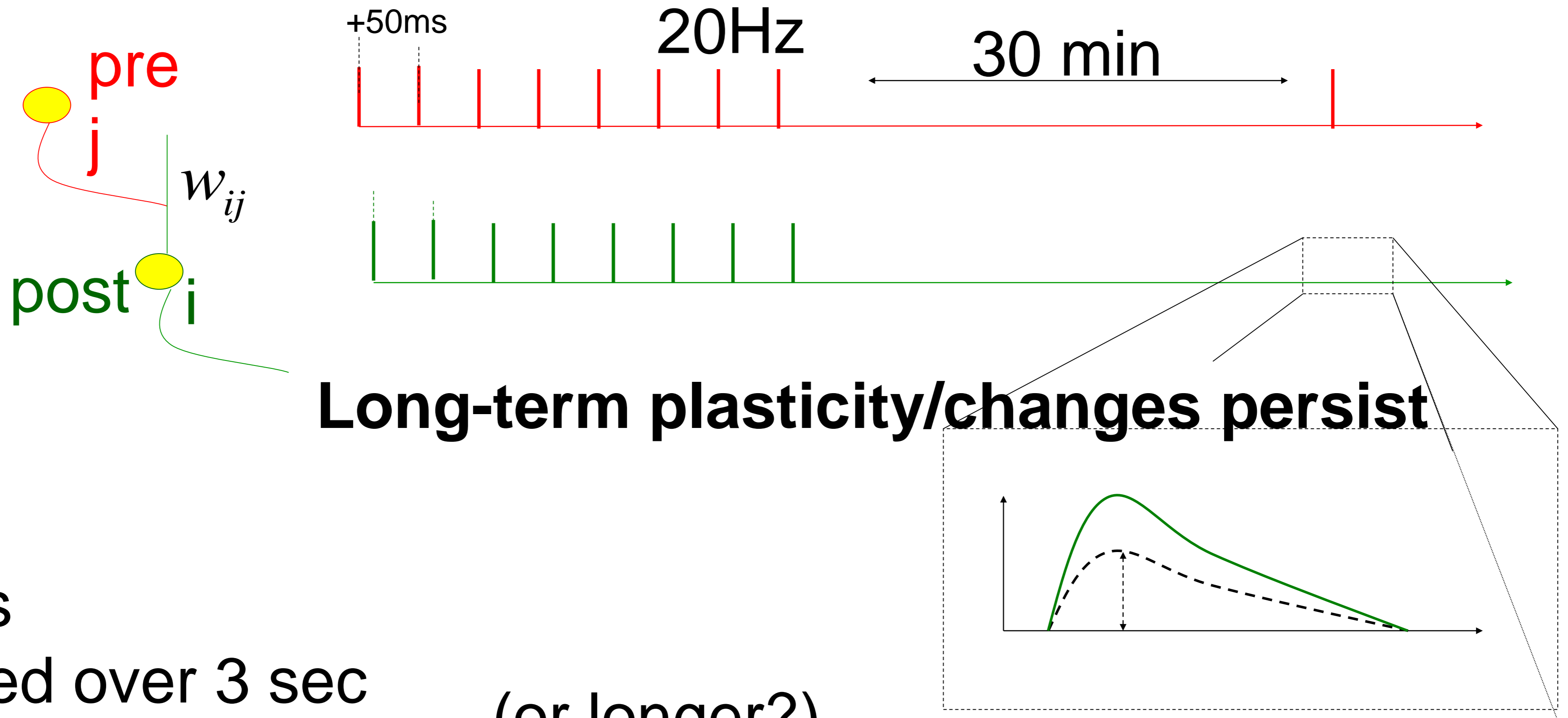
### Changes

- induced over 0.5 sec
- recover over 1 sec

Data: Silberberg, Markram

Fit: Richardson (Tsodyks-Markram model)

# 6.1 Classification of synaptic changes: Long-term plasticity

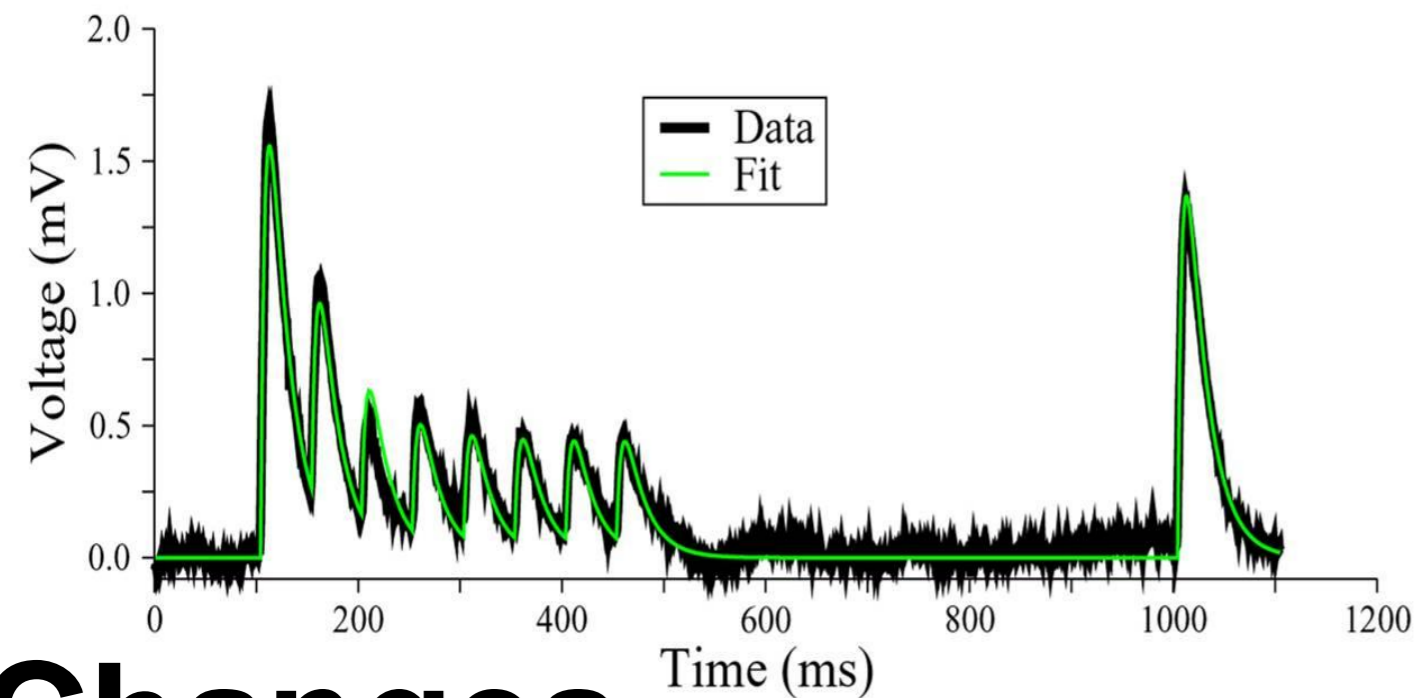


## Changes

- induced over 3 sec (or longer?)
- persist over 1 – 10 hours

# 6.1 Classification of synaptic changes

## Short-Term



## Changes

- induced over 0.1-0.5 sec
- recover over 1 sec

## Protocol

- presynaptic spikes

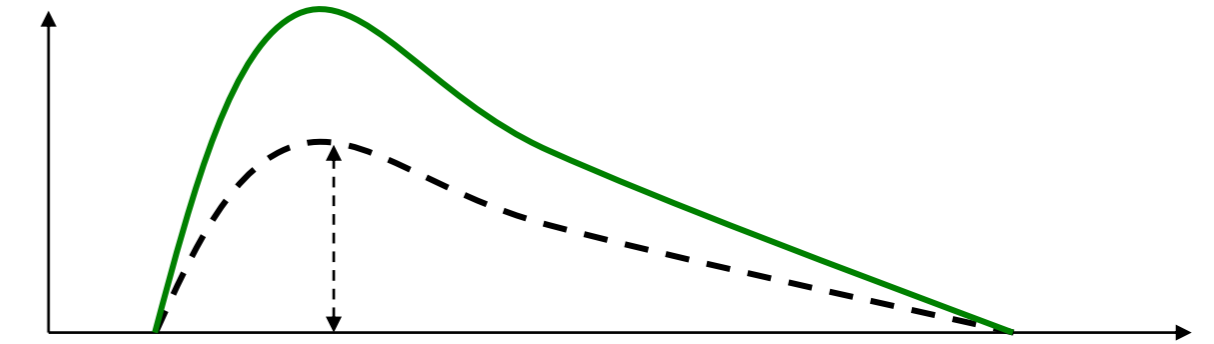
## Model

- well established

(Tosdyks, Senn, Markram)

## vs/ Long-Term

## LTP/LTD/Hebb



## Changes

- induced over 0.5-5sec
- remains over hours

## Protocol

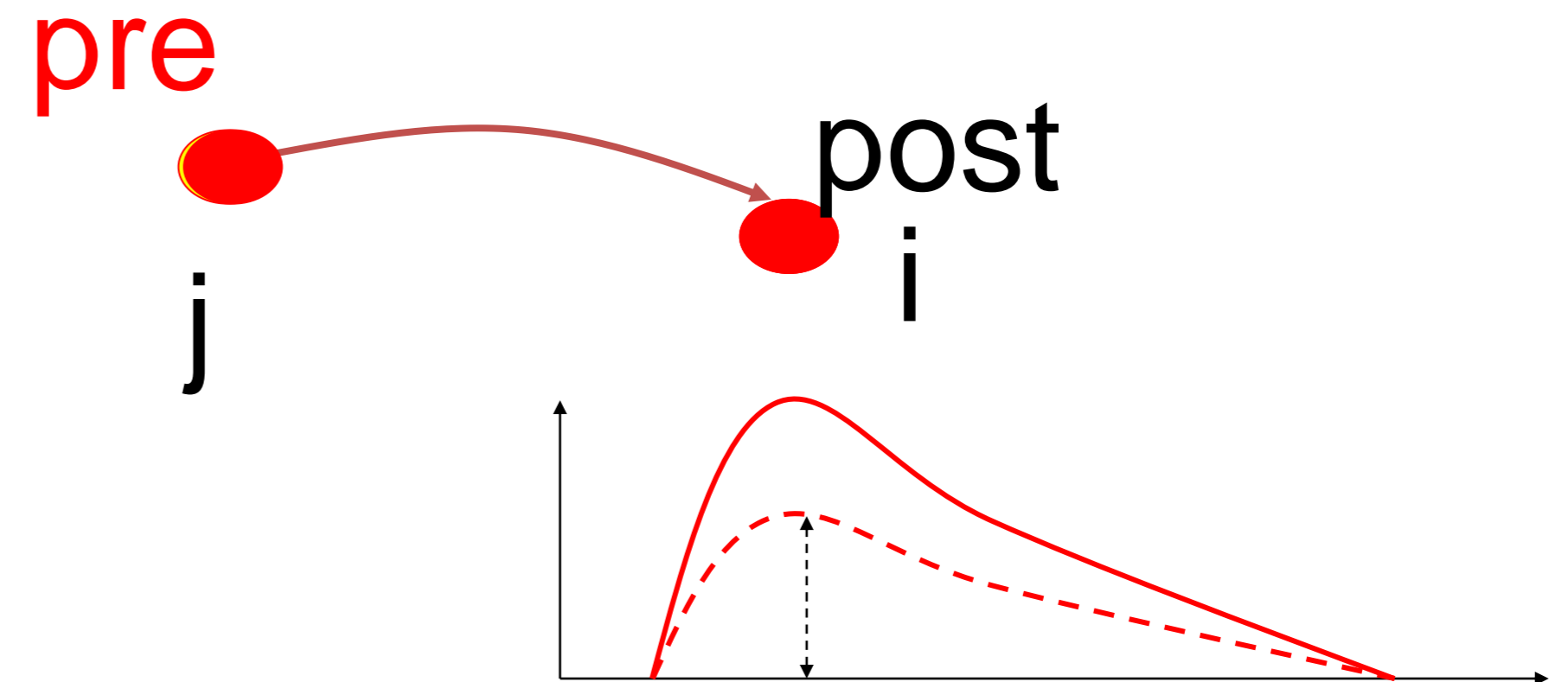
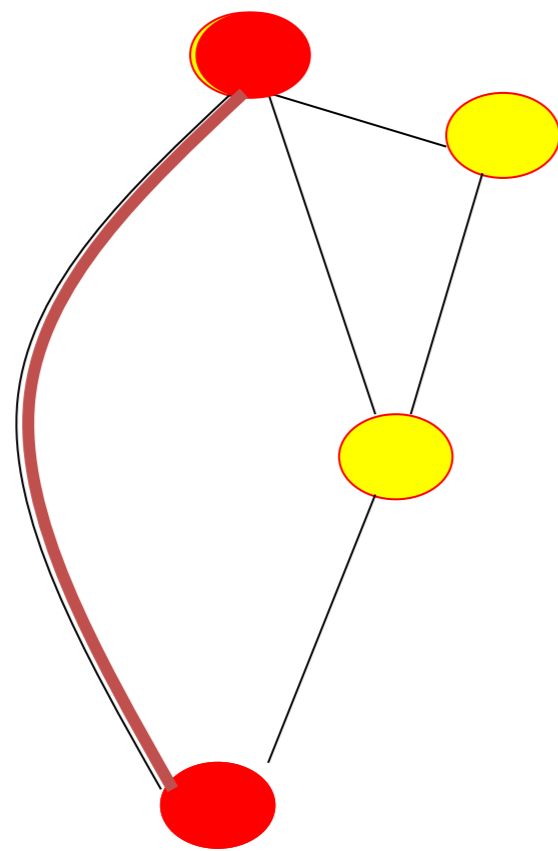
- presynaptic spikes + ...

## Model

- we will see

## 6.1 Classification of synaptic changes: unsupervised learning

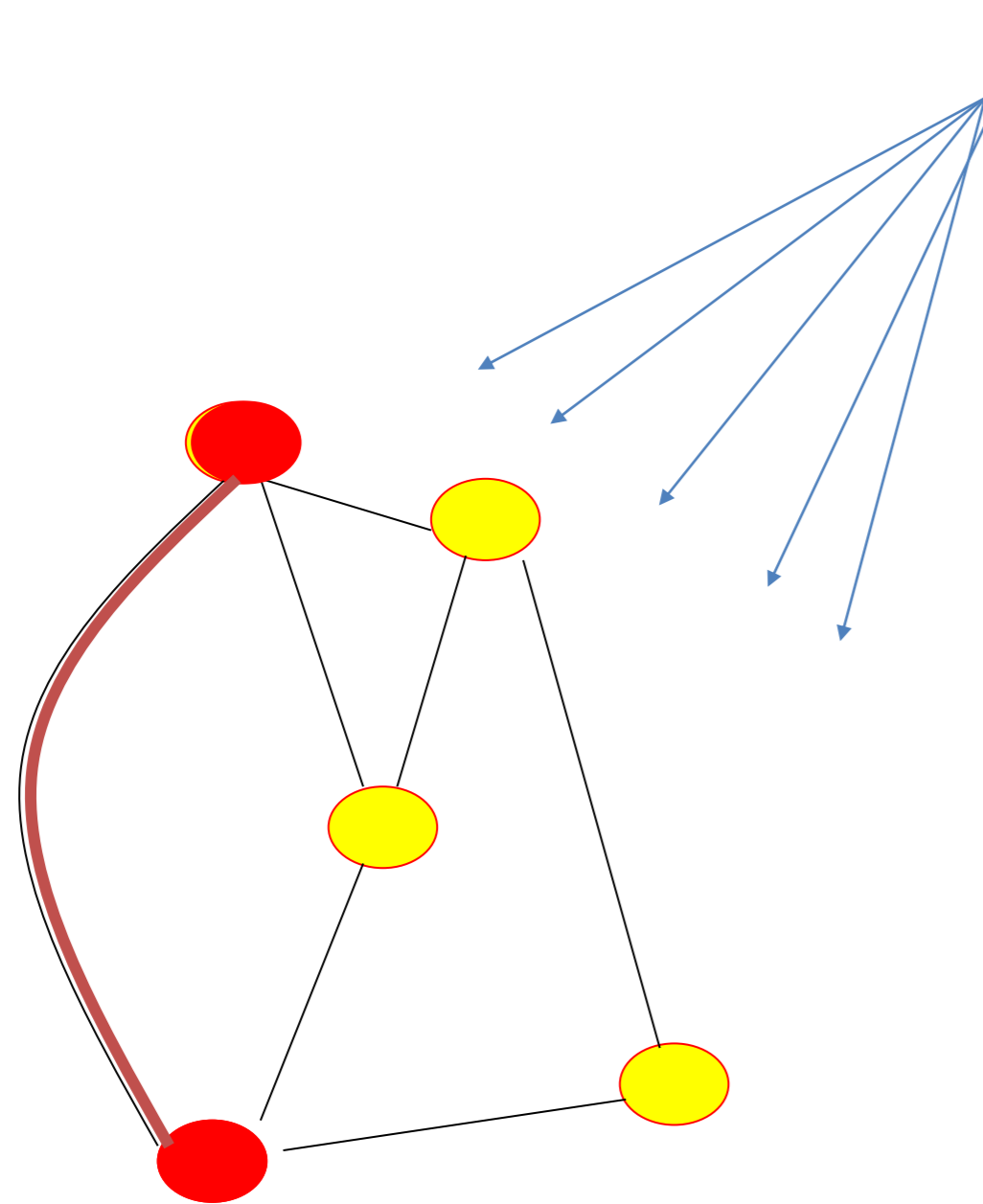
# Hebbian Learning = unsupervised learning



$$w_{ij} \varepsilon \left( -t_j^f \right)$$

$$\Delta w_{ij} \propto F(\text{pre}, \text{post})$$

# 6.1 Classification of synaptic changes: Reinforcement Learning



SUCCESS

**Reinforcement Learning**  
**= reward + Hebb**

$$\Delta w_{ij} \propto F(\underset{\substack{\uparrow \\ \text{local}}}{pre}, \underset{\substack{\uparrow \\ \text{global}}}{post}, \underset{\substack{\uparrow \\ \text{global}}}{SUCCESS})$$

local

global

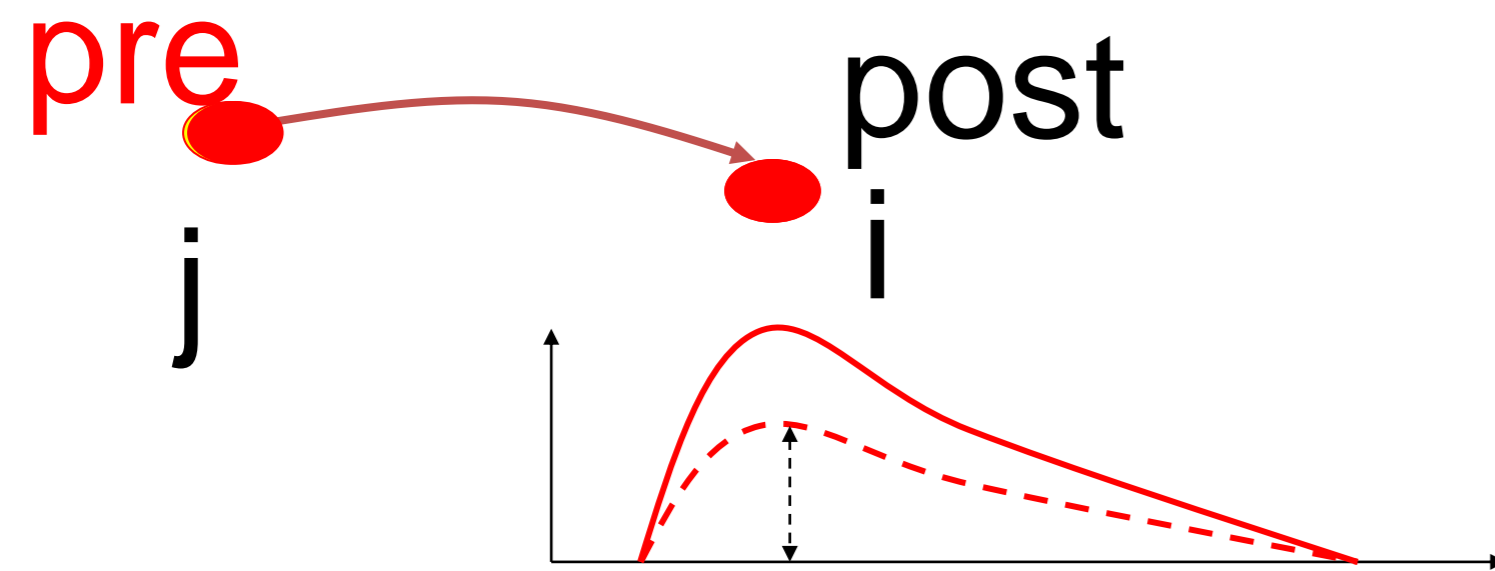


## 6.1 Classification of synaptic changes

# unsupervised vs reinforcement

### LTP/LTD/Hebb Theoretical concept

- passive changes
- exploit statistical correlations

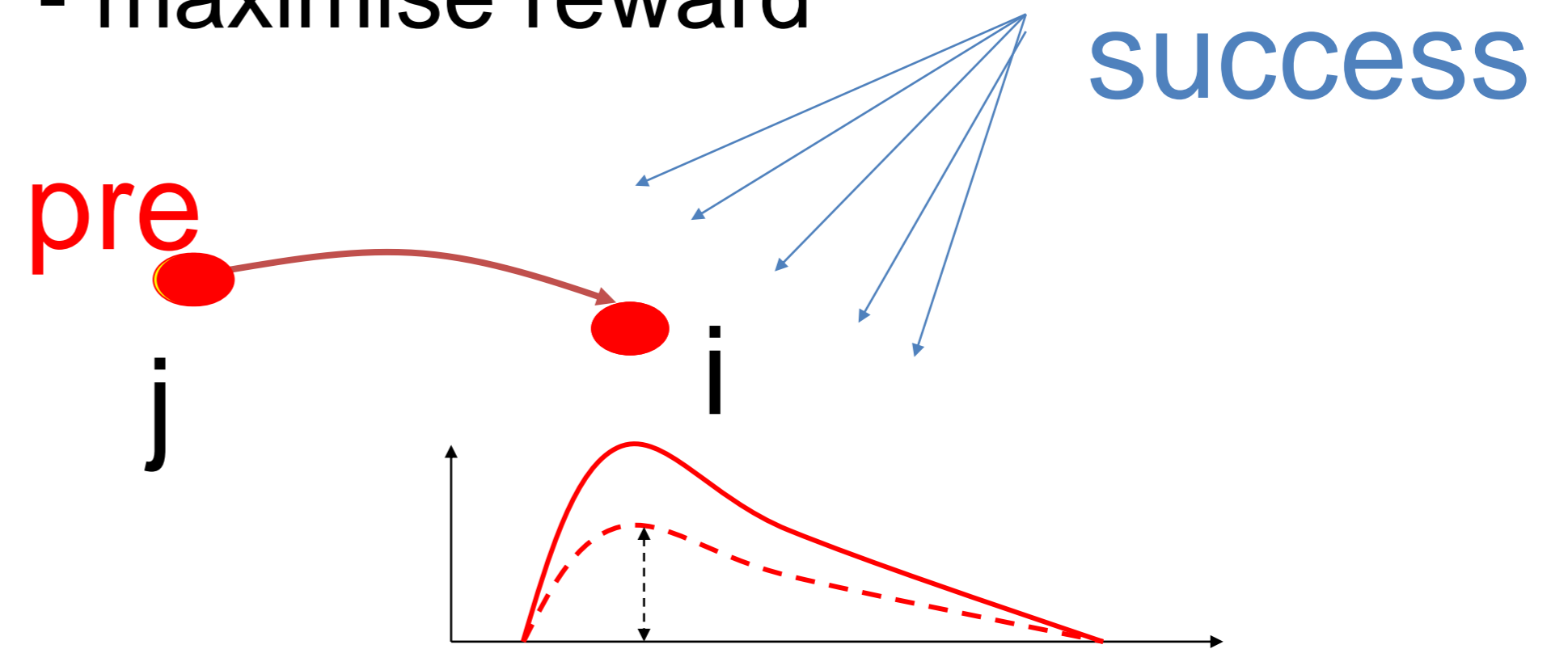


### Functionality

- useful for development  
(wiring for receptive fields)

### Reinforcement Learning Theoretical concept

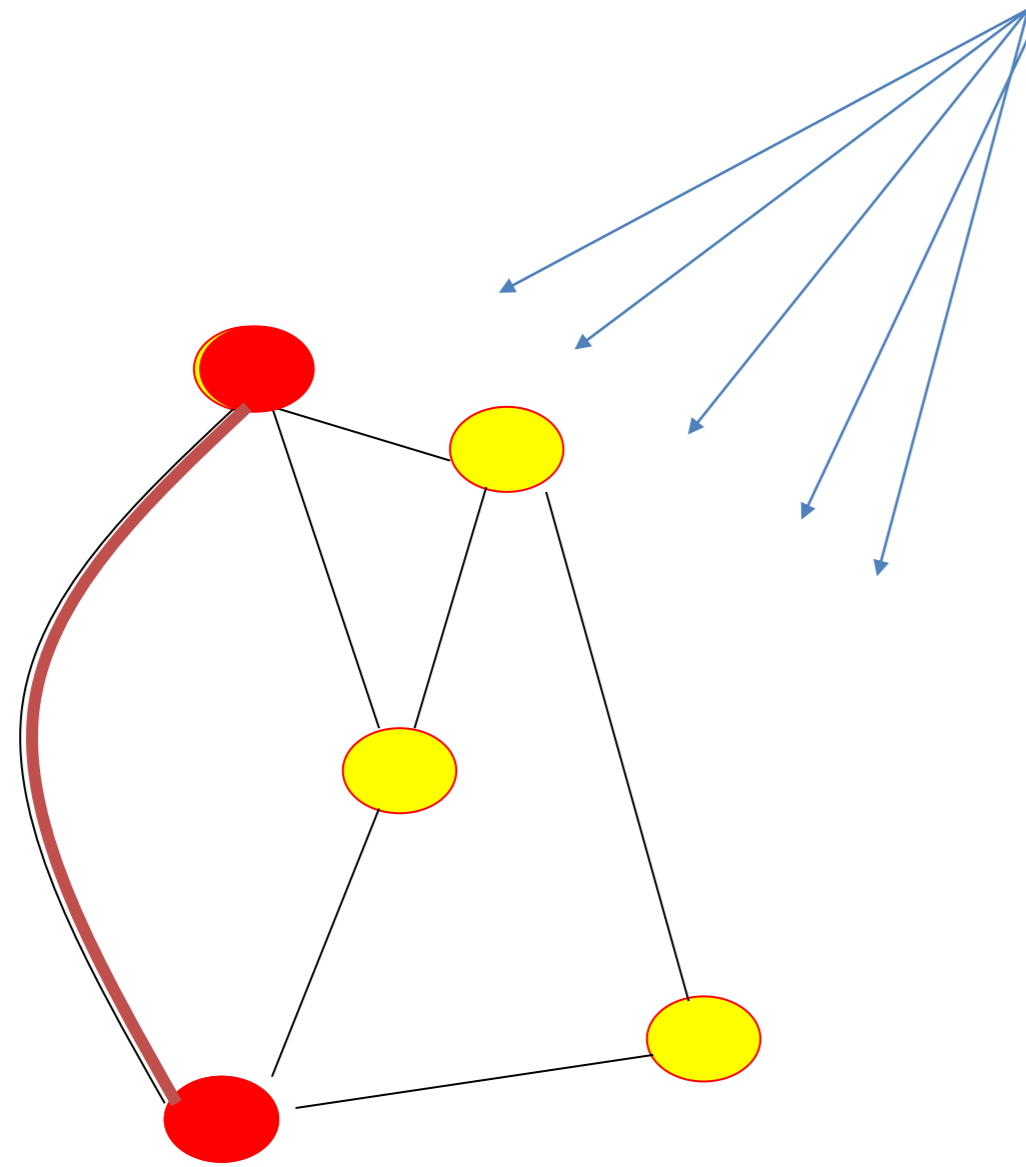
- conditioned changes
- maximise reward



### Functionality

- useful for learning  
a new behavior

# Modulated Hebbian Learning = neuromodulator + Hebb



Neuromodulator: Interestingness, surprise;  
attention; novelty

$$\Delta w_{ij} \propto F(pre, post, MOD)$$

local

global

# Quiz 6.1: Synaptic Plasticity and Learning Rules

## Long-term potentiation

- has an acronym LTP
- takes more than 10 minutes to induce
- lasts more than 30 minutes
- depends on presynaptic activity, but not on state of postsynaptic neuron

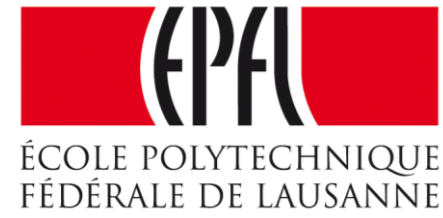
## Short-term potentiation

- has an acronym STP
- takes more than 10 minutes to induce
- lasts more than 30 minutes
- depends on presynaptic activity, but not on state of postsynaptic neuron

## Learning rules

- Hebbian learning depends on presynaptic activity and on state of postsynaptic neuron
- Reinforcement learning depends on neuromodulators such as dopamine indicating reward

# Week 6: Hebbian Learning and Associative Memory



## Biological Modeling of Neural Networks

### Week 6

### Hebbian LEARNING and ASSOCIATIVE MEMORY

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## 6.1 Synaptic Plasticity

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- Short-term Plasticity
- Long-term Plasticity
- Reinforcement Learning

## 6.2 Models of synaptic plasticity

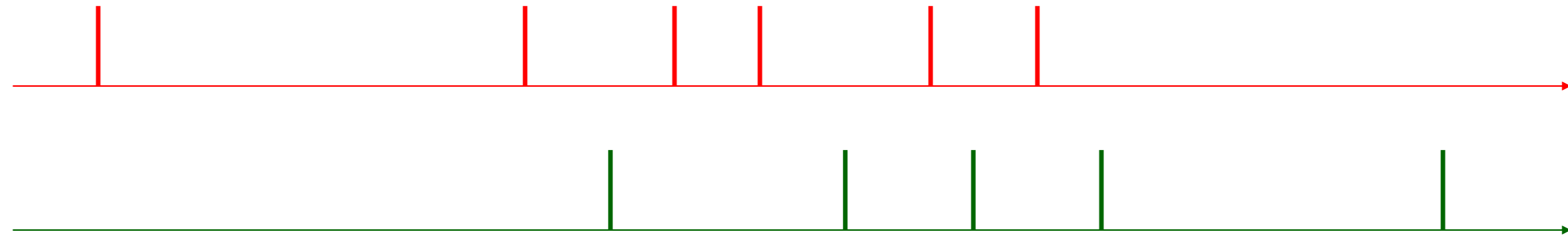
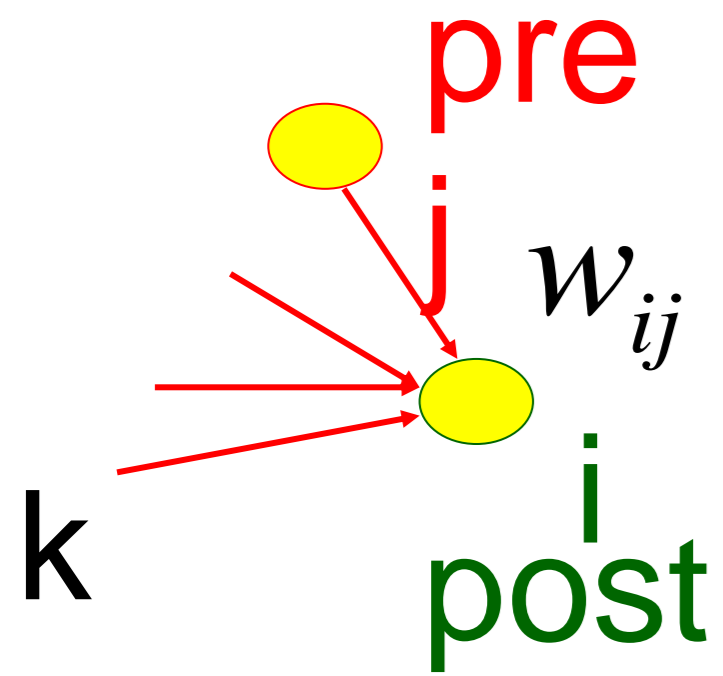
- Hebbian learning rules

## 6.3 Hopfield Model

- probabilistic
- energy landscape

## 6.4 Attractor memories

## 6.2 Hebbian Learning (rate models)



When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then **j**'s efficiency as one of the cells firing **i** is increased

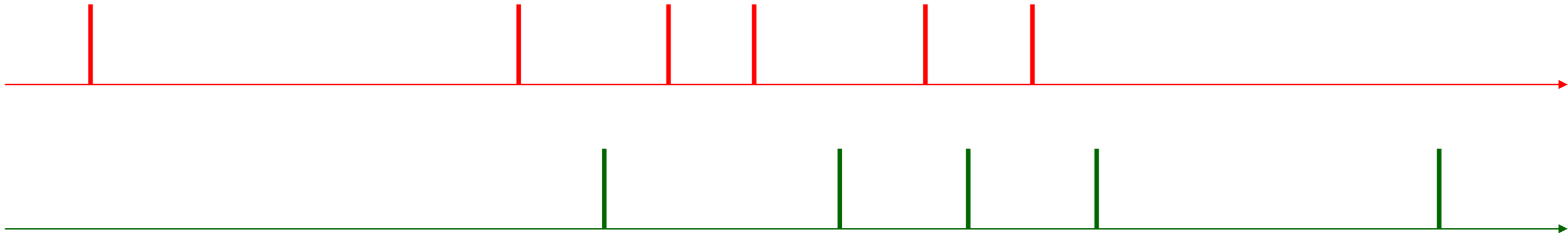
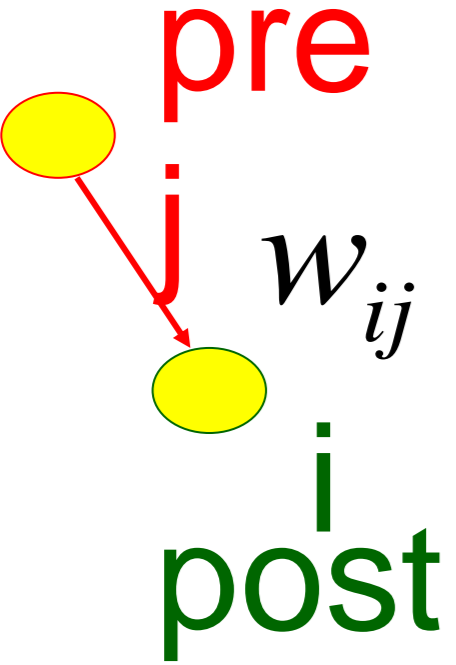
Hebb, 1949

- local rule
- simultaneously active (correlations)

**Rate model:**

**active = high rate = many spikes per second**

# 6.2 Rate-based Hebbian Learning

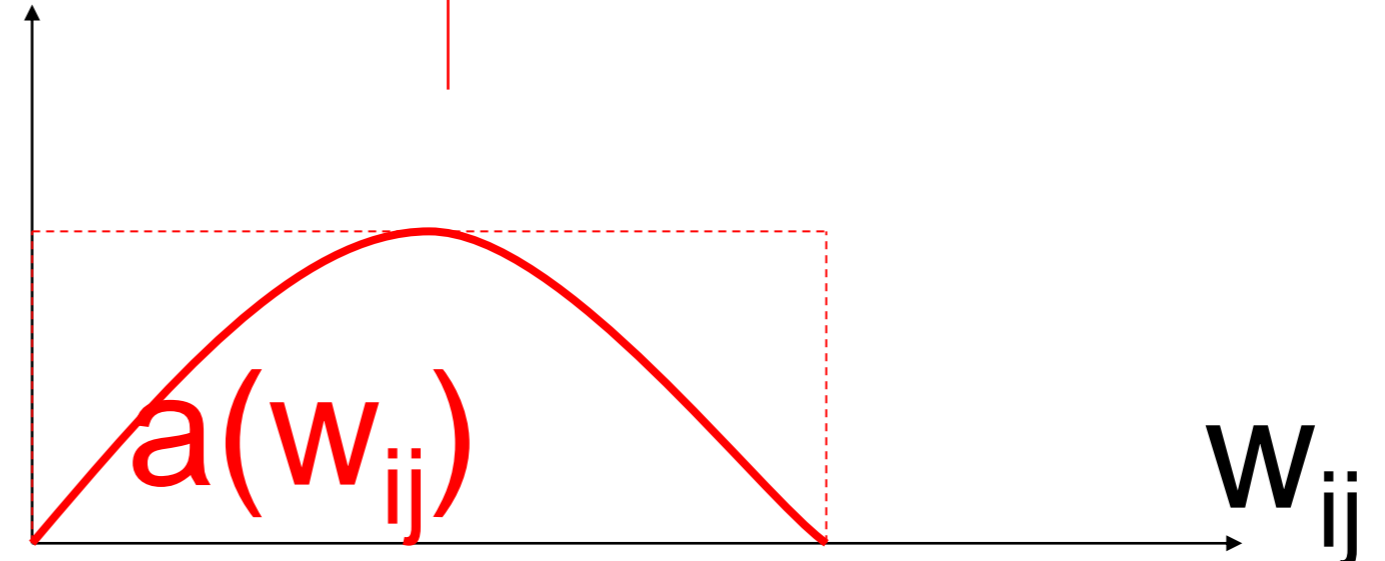


Blackboard

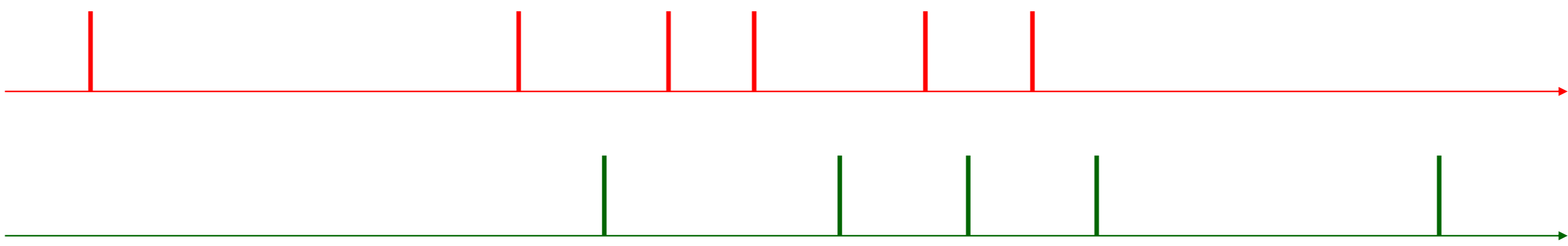
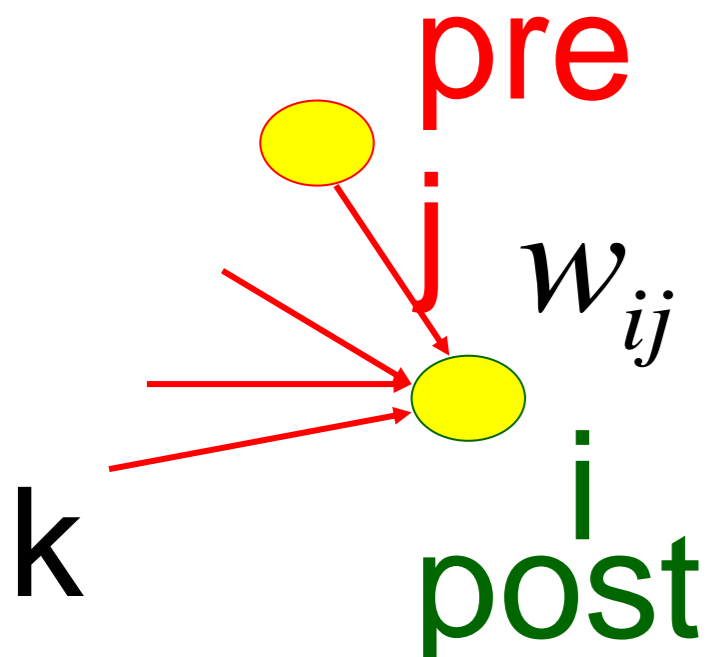
$$\frac{d}{dt} w_{ij} = F(w_{ij}; v_j^{pre}, v_i^{post})$$

$$\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} v_j^{pre} + a_1^{post} v_i^{post} + a_2^{corr} v_j^{pre} v_i^{post} + \dots$$

$a = a(w_{ij})$



# 6.2 Rate-based Hebbian Learning



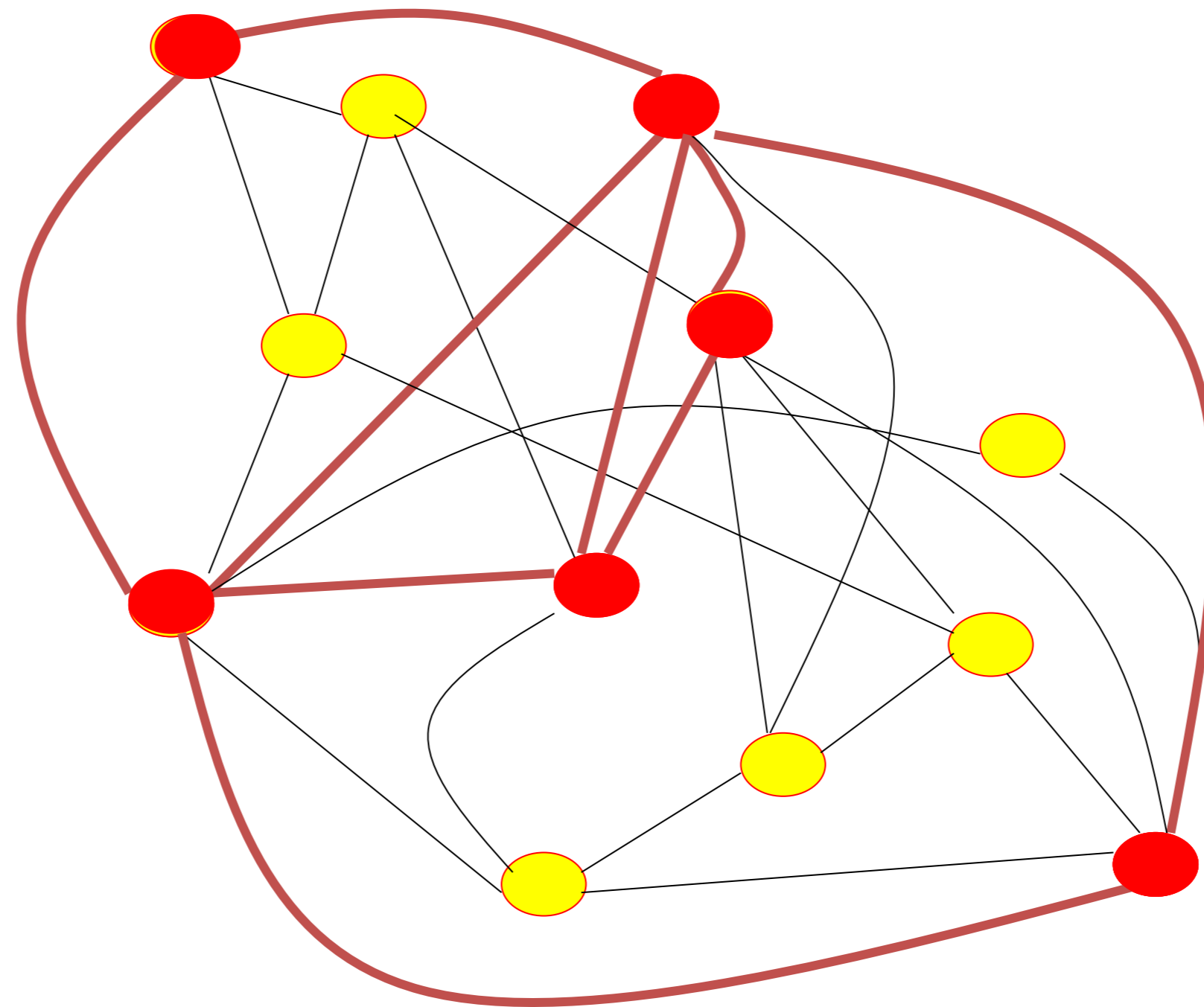
pre  
post

on	off	on	off
on	on	off	off
<b>+</b>	<b>0</b>	<b>0</b>	<b>0</b>

$$\frac{d}{dt} w_{ij} = a_2^{corr} v_j^{pre} v_i^{post}$$

# Hebbian Learning

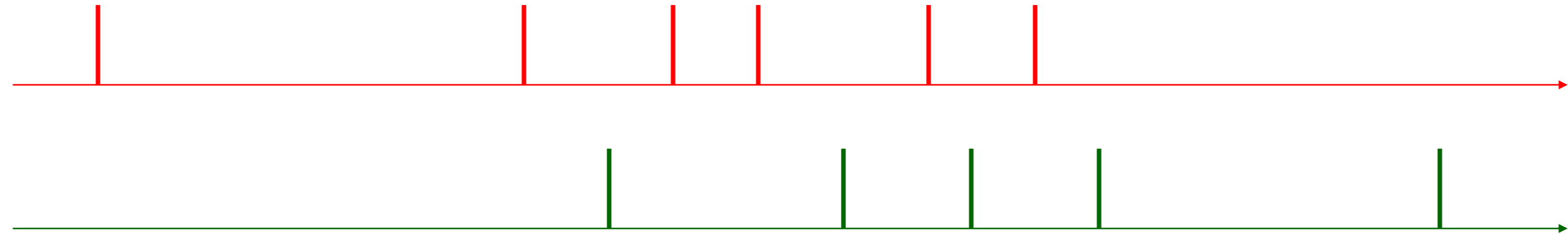
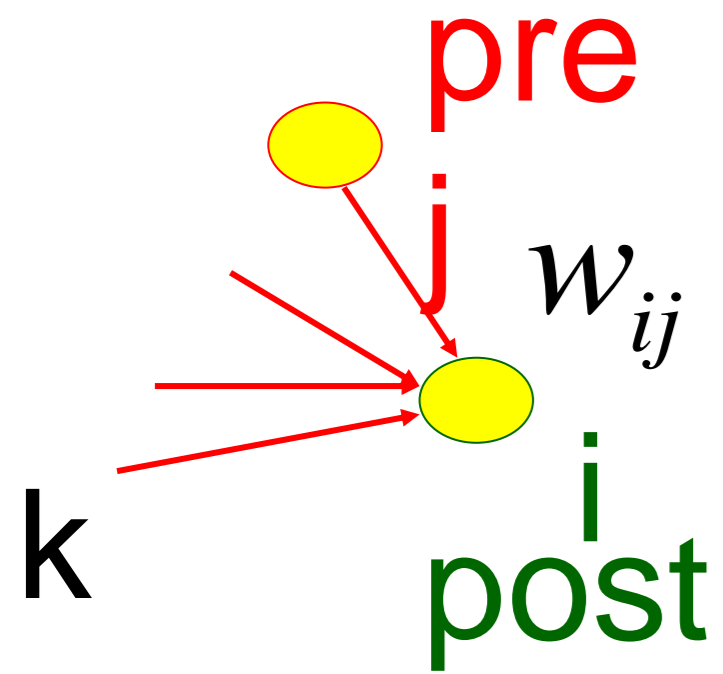
Recall:  
Partial info



item recalled



# 6.2 Rate-based Hebbian Learning



pre  
post

on	off	on	off
on	on	off	off
<b>+</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>+</b>	<b>-</b>	<b>-</b>	<b>-</b>
<b>+</b>	<b>0</b>	<b>-</b>	<b>0</b>
<b>+</b>	<b>-</b>	<b>-</b>	<b>+</b>

$$\frac{d}{dt} w_{ij} = a_2^{corr} v_j^{pre} v_i^{post}$$

$$\frac{d}{dt} w_{ij} = a_2^{corr} v_j^{pre} v_i^{post} - c$$

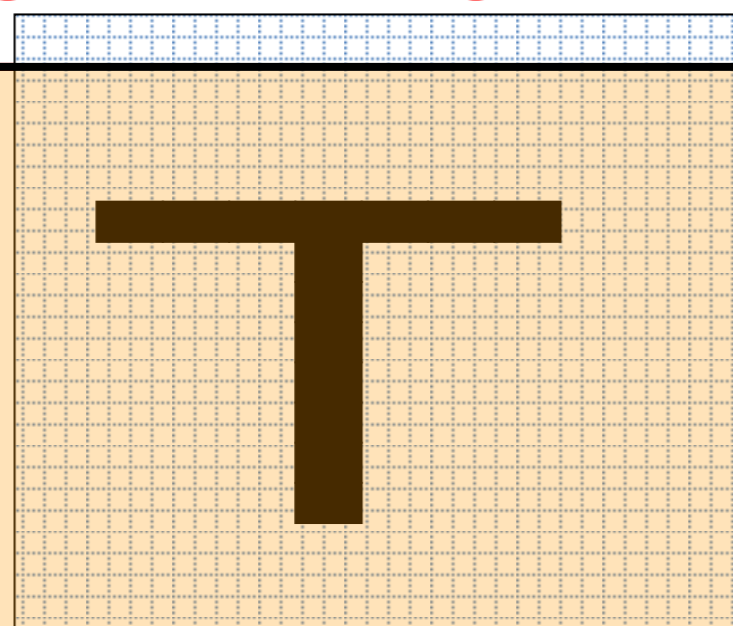
$$\frac{d}{dt} w_{ij} = a_2^{corr} v_j^{pre} (v_i^{post} - \mathcal{G})$$

$$\frac{d}{dt} w_{ij} = a_2^{corr} (v_j^{pre} - \mathcal{G})(v_i^{post} - \mathcal{G})$$

# Exercise 1 now: learning of prototypes

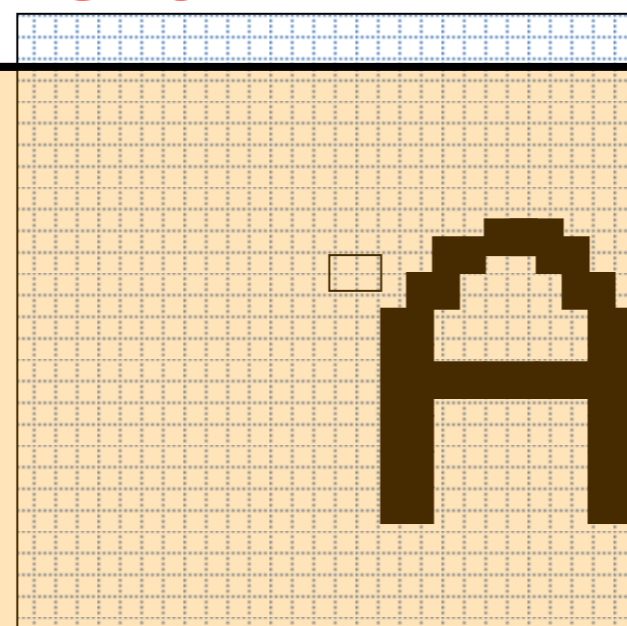
Prototype

$\vec{p}^1$



Prototype

$\vec{p}^2$



interactions

$$(1) \quad w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

a) Show that (1) corresponds to a rate learning rule

$$(2) \quad \frac{d}{dt} w_{ij} = a_2^{corr} (v_j^{pre} - \mathcal{G})(v_i^{post} - \mathcal{G})$$

Next lecture  
10:15

Assume that weights are zero at the beginning;

Each pattern is presented (enforced) during 0.5 sec (One after the other).

note that  $p_j^{\mu} = \pm 1$  but  $v_j \geq 0$

b) Compare with:  $\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} v_j^{pre} + a_1^{post} v_i^{post} + a_2^{corr} v_j^{pre} v_i^{post} + \dots$

c) Is this unsupervised learning?

The end